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ESTIMATION OF C_{pmk} PROCESS CAPABILITY INDEX BASED ON BOOTSTRAP METHOD FOR WEIBULL DISTRIBUTION: A CASE STUDY

Abstract: *The capability indices are widely used by quality professionals as an estimate of process capability. Many process indices have been proposed in last few years. A process capability index* C_{pmk} *is a generalized version of the existing indices confidence interval estimate for* C_{pk} *is defined. A series of* C_p, C_{pk}, C_{pm}. *In this paper the Percentile-t bootstrap* (*PTB*) *simulations using Weibull distribution with different parameters and sample size for the index* C_{pmk} *was undertaken to compare the performance and reliability of bootstrap method and the effect of parameters of Weibull distribution* (*shape and scale*).

Keywords: *Process capability index, Bootstrap sampling,* C_{pmk} *process index, Percentile-t bootstrap method, Weibull distribution*

1. Introduction

Many techniques are available for quality improvement. Statistical Process Control (SPC) is one such TQM technique which is widely accepted for analyzing quality problems and improving the performance of the production process. Mahesh and Prabhuswamy (2010) have illustrated the step by step procedure adopted at a soap manufacturing company to improve the Quality by reducing process variability using Statistical Process Control.

Process capability indices are used to determine whether production process is capable of producing items within a specified tolerance. To evaluate the degree of process capability of system, it is necessary to define a quantitative measure that explains the performance of the system.

A new approach based on noncentral Chi-Square distributions is presented to design the *capability* control charts (Sagbas, 2013).

Process capability analysis were defined as the technique applied in many stages of the product cycle-including process, product design, manufacturing and manufacturing planning, since it help to determine the ability to manufacture parts within the tolerance limits and engineering values (Sagbas, 2009).

Several process capability indices have been proposed to numerically measure whether a process is capable of manufacturing products that meet customer requirements or specifications (Sappakitkamjorn and Niwitpong, 2006). Even though there are many process capability indices, the two

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most commonly used indices are C_p and C_{pk} (Zhang, 2010).

The process indices C_p and C_{pk} have become popular as unitless measures that relate the natural process tolerance (6σ), upper and lower

$$C_p = \frac{USL - LSL}{6\sigma}. \qquad (1)$$

USL and *LSL* denote that the upper and lower specification limits, respectively, for the value of characteristic X to be measured and σ is the process standard deviation. However C_{pk} is more extensively used in practice than C_p because the former considers the degree of process mean, μ, shifted from the center of the specification and the standard deviation, σ, simultaneously. If the process is normally distributed, C_{pk} can be defined as follow (Montgomery, 2011):

$$C_{pk} = \frac{\min(USL - \mu, \mu - LSL)}{3\sigma}. \qquad (2)$$

Since C_{pk} takes on its maximum value when the process is centered between the specification limits, there is natural tendency to adjust the process until μ is located precisely at the midpoint; this however, may not the best location. As result, Hsiang (1985) proposed another index called C_{pm} independently, therefore, the process capability index C_{pm} is called the Taguchi's index or the process capability index based on the loss criterion. The concept of the Taguchi's quality loss has been applied for various quality improvement decisions. (Festervand *et al.*, 2001).

$$C_{pm} = \frac{d}{3(\sigma^2 + (\mu - T)^2)^{\frac{1}{2}}}, \qquad (3)$$

where $d = \frac{1}{2}(USL - LSL)$ and T is target value.

Pearn and Shu (2003) have applied the C_{pm} control chart to the practical production environment of precision electronic devices. As a result, they have verified the effectiveness of the C_{pm} control chart and concluded that the approach is useful for quality improvement decisions.

A new index C_{pmk} is proposed by Pearn et al (1992), C_{pmk} is generalized of C_{pm} and C_{pk}.

$$C_{pmk} = \frac{\min(USL - \mu, \mu - LSL)}{3(\sigma^2 + (\mu - T)^2)^{\frac{1}{2}}}. \qquad (4)$$

In actual practice the value of μ and σ are not known and $\overline{X}$ and S (the sample mean and σ sample standard deviation respectively) are used as estimators for μ and σ. This then gives

$$\hat{C}_{pmk} = \frac{\min(USL - \overline{X}, \overline{X} - LSL)}{3(S^2 + (\overline{X} - T)^2)^{\frac{1}{2}}}. \qquad (5)$$

This expression, (5), can be rewritten as

$$\hat{C}_{pmk} = \frac{d - \left|\overline{X} - M\right|}{3(\frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2 + (\overline{X} - T)^2)^{\frac{1}{2}}}, \qquad (6)$$

where $M = \frac{USL + LSL}{2}$.

2. The Bootstrap Method

The bootstrap is a computer-based and resampling method for assigning measures of accuracy to statistical estimates.

Let $X_1, X_2, \ldots, X_n$ be a sample of X taken from a process, i.e., sequence of n i.i.d. random variables $(X_1, X_2, \ldots, X_n \sim F)$. A bootstrap sample is one of size n drawn (with replacement) from the original sample and is denoted by, $X_1^*, X_2^*, \ldots, X_n^* \sim F$ (Tosasukul *et al*., 2009).

There are a total of n^n such resamples possible. In our case, these resamples would then be used to calculate n^n values (each) of

$\hat{C}^*$. (C denoted PCI). Each of these would be an estimate of $\hat{C}$ and the entire collection would constitute the complete bootstrap distribution for $\hat{C}$.

Bootstrap sampling is equivalent to sampling with replacement form the empirical probability distribution. Thus, the bootstrap distribution of $\hat{C}$ is an estimate of the distribution of C. In practice, a rough minimum of 1000 bootstrap resamples are usually sufficient to compute reasonably accurate confidence interval estimates (Efron and Tibshirani, 1986). Therefore, it is assumed that *B=1000* times bootstrap samples are taken and *B=1000* times bootstrap $\hat{C}$ are calculated and ordered from smallest to largest.

In the following we present three possible constructions for the confidence interval of C_{pmk} using bootstrap techniques.

2.1. Standard Bootstrap (SB) Confidence Interval

From the B=1000 bootstrap estimates $\hat{C}_i^*$, for i = 1, 2, …, B, calculate the sample average $\bar{C}^*$ and the sample standard deviation $S_c^* = \sqrt{\frac{1}{B-1}\sum(\hat{C}_i^* - \bar{C}^*)^2}$. The standard bootstrap confidence interval for C is

$$\left[\hat{C} - Z_\alpha S_c^*,\ \hat{C} + Z_\alpha S_c^*\right]. \tag{7}$$

2.2. Percentile Bootstrap (PB) Confidence Interval

From the ordered collection of $\hat{C}_i^*$, for i=1,2,…,n, the percentile bootstrap confidence interval for C is

$$\left[\hat{C}^*(\alpha\beta), \hat{C}^*((1-\alpha)\beta)\right]. \tag{8}$$

2.3. Percentile-t Bootstrap (PTB) Confidence Interval

There are three cases of confidence intervals for C_{pmk} (Choi *et al.*, 1995). For the case of M>μ, confidence interval for C_{pmk} is

$$\left[\hat{C}_{pmk} - m^{\frac{1}{2}}\hat{\sigma}_{pmk}\hat{\vartheta}_{pmk,\frac{1+\alpha}{2}},\ \hat{C}_{pmk} - m^{\frac{1}{2}}\hat{\sigma}_{pmk}\hat{\vartheta}_{pmk,\frac{1-\alpha}{2}}\right] \tag{9}$$

where

$$\hat{\sigma}_{pmk}^2 = \frac{1}{9}[S^2 + (T-\bar{X})^2]^{-3}$$
$$\times[S^2 + (T-\bar{X})^2 + (T-\bar{X})(d+\bar{X}-M)]^2$$
$$-(d+\bar{X}-M)\hat{\mu}_3[S^2 + (T-\bar{X})^2 + (T-\bar{X})$$
$$\times(d+\bar{X}-M)]. \tag{10}$$

and $\hat{\vartheta}_{pmk,\alpha}$ is defined as follows:

$$P\left(\frac{\sqrt{m}(\hat{C}_{pmk}^* - \hat{C}_{pmk})}{\hat{\sigma}_{pmk}^*} \le \hat{\vartheta}_{pmk,\alpha}\right) = \alpha \tag{11}$$

$\hat{C}_{pmk}^*$ and $\hat{\sigma}_{pmk}^*$ are the resample version of $\hat{C}_{pmk}$ and $\hat{\sigma}_{pmk}$ respectively.

For the case M < μ, the $\hat{\sigma}_{pmk}^2$ is different,

$$\hat{\sigma}_{pmk}^2 = \frac{1}{9}[S^2 + (T-\bar{X})^2]^{-3}$$
$$\times[S^2 + (T-\bar{X})^2 - (T-\bar{X})(d-\bar{X}+M)]^2$$
$$-(d-\bar{X}+M)\hat{\mu}_3[S^2 + (T-\bar{X})^2]$$
$$-(T-\bar{X})(d-\bar{X}+M)] + \frac{1}{4}(d-\bar{X}+M)^2$$
$$\times(\hat{\mu}_4 - S^4)). \tag{12}$$

For the case of M=μ, C_{pmk} is reduced to C_{pm} and

thus we have the same confidence interval as for C_{pm}, which is given as

$$\left[\hat{C}_{pm} - m^{\frac{1}{2}}\hat{\sigma}_{pm}\hat{\vartheta}_{pm,\frac{1+\alpha}{2}},\ \hat{C}_{pm} - m^{\frac{1}{2}}\hat{\sigma}_{pm}\hat{\vartheta}_{pm,\frac{1-\alpha}{2}}\right] \tag{13}$$

where

$$\hat{C}_{pm} = \frac{d}{3\sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2}},$$

$$\hat{\sigma}^2_{pm} = \frac{USL - LSL}{3\sigma(S^2 + (T - \bar{X})^2)^3}$$

$$\times(S^2(T - \bar{X})^2 - (T - \bar{X})\hat{\mu}_3 + \frac{1}{4}(\hat{\mu}_4 - S^4)) \cdot$$

and $\hat{\vartheta}_{pm,\alpha}$ is defined as follows:

$$P(\frac{\sqrt{m}(\hat{C}^*_{pm} - \hat{C}_{pm})}{\hat{\sigma}^*_{pm}} \le \hat{\vartheta}_{pm,\alpha}) = \alpha \qquad (14)$$

To study the different confidence intervals, their estimated coverage probabilities and average widths are considered. For each of the methods considered, a $(1-\alpha)100\%$ confidence interval denoted by (L, U) is obtained (based on $N = 10000$ replicates). The estimated coverage probability and the average width are given by (Panichkitkosolkul, 2013):

$$\text{Coverage Probability} = \frac{\#(L \le C_{pk} \le U)}{N},$$

and

$$\text{Average Width} = \frac{\sum_{i=1}^{N}(U_i - L_i)}{N}.$$

Panichkitkosolkul and Saothayanun (2012) concerned the construction of bootstrap confidence intervals for the process capability index in the case of half-logistic distribution.

3. Weibull Distribution

Waloddi Weibull (1887-1979) was a Swedish engineer and scientist well-known for his work on strength of materials and fatigue analysis. The pdf of Weibull distribution, also known as the *Extreme Value Type III* distribution, is given as follows:

$$f_{a,b}(x) = ba^{-b}x^{b-1}e^{-(\frac{x}{a})^b} I_{(0,\infty)}(x) \cdot \qquad (15)$$

The Weibull distribution is defined for *x>0*, and both distribution parameters (*a*-scale, *b*-shape) are positive.

Even though the Weibull distribution was originally developed to address the problems arising in material sciences, it is widely used in many other areas thanks to its flexibility. When *b*=1, this distribution reduces to the Exponential model, and when *b*=2, it mimics the Rayleigh distribution which is mainly used in telecommunications. In addition, it resembles the Normal distribution when *b*=3.5.

If $X \sim$ WEIBULL(*a*,*b*) then the mean and variance of X are given as follows

$$E_{a,b}(X) = a[\Gamma(1+b^{-1})],$$

$$Var_{a,b}(X) = a^2[\Gamma(1+2b^{-1}) - \Gamma^2(1+b^{-1})],$$

4. Simulation Study

Here we want to compare the new Percentile–t bootstrap confidence interval with Standard and Percentile confidence interval and the effect of the Weibull's parameters in performance of bootstrap method. For this purpose, we choose the following values given in Franklin and Wasserman (1992): *USL*=61, *LSL*=40, *T*=49, μ=50, σ=2. This values yield C_{pmk}=1.491, which represent a capable process.

We chose the resample size *m* = 10, 20, 30 (for the *n*=10), *m* = 30, 40, 50 (for the *n*=30) and *m* = 40, 50, 60 (for the *n* = 40). B = 1000 bootstrap resamples (each of size *m*) drawn from each sample of size *n* and a 90% bootstrap confidence interval is constructed for each method. It is then determined if the calculated confidence interval for each type contains the true index value. The mean and standard error of the length of each confidence interval is also evaluated. This single simulation was then replicated *N*=1000 times and, thus, a proportion of times that the true value of the index is contained within the calculated interval was calculated as well as an average length, and a standard error of average length of the 90% confidence intervals. In order to make each distribution to have the specified values of, μ =50 and σ =2, we transform Weibull distribution as follows:

X $\sim$ WEIBULL(a,b)

$$\xrightarrow{\eta=mean(X),S=Sd(X)} \sigma(\frac{X-\eta}{S})+\mu,$$

For example :

X $\sim$ WEIBULL(1,1)

$$\xrightarrow{\eta=1,S=1} 2(\frac{X-1}{1})+50,$$

X $\sim$ WEIBULL(2,3)

$$\xrightarrow{\eta=1.786,S=0.6491} 2(\frac{X-1.786}{0.6491})+50,$$

5. Simulation results

The simulation results are tabulated in Tables 1-6. For a Weibull distribution process with parameters (*a*,*b*) when *b* is less than 3 (Tables 1,2,3), standard method provide higher coverage probability for all size and shorter standard error for $n \geq 30$. But the confidence interval of the Percentile-t method is shorter for all size and provides shorter standard error for $n \leq 3$. In general case, Percentile method has the lower coverage probability. The mean of the Percentile-t method is less than two other methods for all size.

Table 1. Coverage Probability, Mean, Standard Error of the Width for the 90% Bootstrap Confidence Interval –Weibull (1,2)

C_{pmk}		n=10			n=30			n=40		
wbl(1,2)	m	10	20	30	30	40	50	40	50	60
PTB	**Cov**	**0.817**	**0.638**	**0.531**	**0.859**	**0.804**	**0.767**	**0.887**	**0.847**	**0.798**
	Mean	1.0601	0.7003	0.5414	0.64	0.5453	0.4837	0.5528	0.4887	0.4423
	S.E.	0.3914	0.2562	0.1612	0.2187	0.181	0.1556	0.147	0.1263	0.1133
SB	**Cov**	**0.874**	**0.699**	**0.593**	**0.871**	**0.818**	**0.783**	**0.891**	**0.847**	**0.8**
	Mean	1.3215	0.8049	0.6234	0.6771	0.5727	0.504	0.5744	0.5064	0.4567
	S.E.	0.604	0.3217	0.2295	0.179	0.1513	0.1309	0.1286	0.113	0.1015
PB	**Cov**	**0.781**	**0.623**	**0.525**	**0.819**	**0.762**	**0.724**	**0.842**	**0.801**	**0.764**
	Mean	1.2127	0.7822	0.6014	0.6629	0.5635	0.4978	0.5661	0.4998	0.4521
	S.E.	0.519	0.3131	0.2046	0.171	0.1474	0.1297	0.1264	0.1106	0.0995

Table 2. Coverage Probability, Mean, Standard Error of the Width for the 90% Bootstrap Confidence Interval –Weibull (3,2)

C_{pmk}		n=10			n=30			n=40		
wbl(3,2)	m	10	20	30	30	40	50	40	50	60
PTB	**Cov**	**0.799**	**0.623**	**0.514**	**0.855**	**0.791**	**0.736**	**0.889**	**0.852**	**0.812**
	Mean	1.0675	0.7104	0.545	0.6438	0.5499	0.4879	0.5435	0.4828	0.4374
	S.E.	0.3951	0.286	0.1784	0.2062	0.1729	0.1509	0.1289	0.118	0.1004
SB	**Cov**	**0.876**	**0.679**	**0.563**	**0.871**	**0.804**	**0.755**	**0.897**	**0.857**	**0.814**
	Mean	1.3492	0.8147	0.6298	0.6856	0.5796	0.5106	0.569	0.5029	0.4538
	S.E.	0.6457	0.3433	0.2442	0.1851	0.1573	0.135	0.1168	0.102	0.0923

PB	**Cov**	**0.766**	**0.61**	**0.494**	**0.797**	**0.757**	**0.701**	**0.853**	**0.794**	**0.754**
	Mean	1.2346	0.7914	0.6068	0.671	0.5699	0.5039	0.5605	0.4971	0.4488
	S.E.	0.5647	0.334	0.218	0.1775	0.1519	0.1339	0.1147	0.1002	0.091

Table 3. Coverage Probability, Mean, Standard Error of the Width for the 90% Bootstrap Interval –Weibull (4,1)

C_{pmk}		*n*=10			*n*=30			*n*=40		
wbl(4,1)	*m*	10	20	30	30	40	50	40	50	60
PTB	***Cov***	***0.709***	***0.562***	***0.435***	***0.813***	***0.757***	***0.709***	***0.852***	***0.821***	***0.778***
	Mean	1.601	1.114	0.7725	1.188	0.9931	0.8697	1.0451	0.9139	0.807
	S.E.	0.9699	0.7482	0.4175	0.7593	0.6027	0.5222	0.5803	0.488	0.423
SB	***Cov***	***0.853***	***0.672***	***0.552***	***0.871***	***0.805***	***0.75***	***0.881***	***0.836***	***0.786***
	Mean	2.5766	1.4787	1.0739	1.1999	0.9898	0.8565	1.0093	0.8738	0.7776
	S.E.	1.2568	0.7454	0.4941	0.4315	0.3601	0.3081	0.3232	0.2813	0.2469
PB	***Cov***	***0.643***	***0.514***	***0.433***	***0.746***	***0.691***	***0.647***	***0.786***	***0.753***	***0.722***
	Mean	2.3046	1.4173	0.9474	1.1406	0.9531	0.8318	0.9686	0.8461	0.7582
	S.E.	1.1146	0.7464	0.3873	0.392	0.3381	2963	0.3023	0.2657	0.2388

For b≥3 (Tables 4, 5, 6), Percentile-t method provides higher coverage probability when *n*≥30 and shorter confidence interval and standard error for all size. For *n*≤30 standard method provide higher coverage probability, but the confidence interval and standard error is higher than Percentile-t method. Percentile method has the lower coverage probability. The mean of the Percentile-t method is less than two other methods for all size.

Table 4. Coverage Probability, Mean, Standard Error of the Width for the 90% Bootstrap Confidence Interval –Weibull (1,3)

C_{pmk}		*n*=10			*n*=30			*n*=40		
wbl(1,3)	*m*	10	20	30	*30*	*40*	*50*	*40*	*50*	*60*
PTB	***Cov***	***0.883***	***0.706***	***0.595***	***0.91***	***0.843***	***0.788***	***0.907***	***0.863***	***0.828***
	Mean	1.035	0.6559	0.5063	0.5544	0.4708	0.4169	0.4739	0.418	0.378
	S.E.	0.3553	0.2188	0.1425	0.1185	0.0999	0.0886	0.0879	0.0782	0.0713
SB	***Cov***	***0.895***	***0.73***	***0.626***	***0.899***	***0.836***	***0.782***	***0.903***	***0.862***	***0.821***
	Mean	1.1471	0.7095	0.5541	0.5679	0.4822	0.4259	0.4806	0.4243	0.3837
	S.E.	0.5393	0.2908	0.2051	0.1247	0.1068	0.0939	0.092	0.0815	0.0737
PB	***Cov***	***0.828***	***0.685***	***0.583***	***0.864***	***0.81***	***0.771***	***0.875***	***0.841***	***0.802***
	Mean	1.0624	0.6922	0.5394	0.5583	0.4758	0.4216	0.475	0.4199	0.3801
	S.E.	0.4531	0.284	0.1799	0.1237	0.1043	0.0926	0.0902	0.806	0.756

Table 5. Coverage Probability, Mean, Standard Error of the Width for the 90% Bootstrap Confidence Interval –Weibull (3,3)

C_{pmk}		*n=10*			*n=30*			*n=40*		
wbl(3,3)	*m*	*10*	*20*	*30*	*30*	*40*	*50*	*40*	*50*	*60*
PTB	***Cov***	***0.861***	***0.692***	***0.572***	***0.904***	***0.845***	***0.793***	***0.912***	***0.875***	***0.821***
	Mean	*1.0294*	*0.6583*	*0.5104*	*0.5621*	*0.477*	*0.4214*	*0.4784*	*0.4224*	*0.3822*
	S.E.	*0.3645*	*0.2294*	*0.1538*	*0.1208*	*0.1021*	*0.09*	*0.0977*	*0.0853*	*0.0775*
SB	***Cov***	***0.88***	***0.702***	***0.6***	***0.909***	***0.842***	***0.797***	***0.906***	***0.866***	***0.809***
	Mean	*1.1481*	*0.713*	*0.5567*	*0.5749*	*0.4869*	*0.4307*	*0.4849*	*0.429*	*0.3879*
	S.E.	*0.525*	*0.2798*	*0.2021*	*0.1307*	*0.1103*	*0.0965*	*0.0974*	*0.0859*	*0.0783*
PB	***Cov***	***0.807***	***0.667***	***0.546***	***0.873***	***0.825***	***0.779***	***0.853***	***0.82***	***0.778***
	Mean	*1.0585*	*0.6924*	*0.5432*	*0.5645*	*0.4809*	*0.4258*	*0.479*	*0.4243*	*0.3842*
	S.E.	*0.4368*	*0.2671*	*0.1885*	*0.126*	*0.107*	*0.0945*	*0.0951*	*0.0846*	*0.0773*

Table 6. Coverage Probability, Mean, Standard Error of the Width for the 90% Bootstrap Confidence Interval –Weibull (4,3)

C_{pmk}		*n*=10			*n*=30			*n*=40		
wbl(4,3)	*m*	10	20	30	*30*	*40*	*50*	*40*	*50*	*60*
PTB	***Cov***	***0.866***	***0.675***	***0.569***	***0.9***	***0.842***	***0.786***	***0.906***	***0.855***	***0.812***
	Mean	1.0629	0.6746	0.5193	0.5537	0.47	0.4157	0.4753	0.4189	0.3799
	S.E.	0.4213	0.2429	0.1549	0.1196	0.0996	0.0872	0.0938	0.0826	0.074
SB	***Cov***	***0.885***	***0.701***	***0.597***	***0.892***	***0.832***	***0.787***	***0.896***	***0.849***	***0.811***
	Mean	1.2197	0.7394	0.5759	0.5636	0.4788	0.4239	0.4811	0.4246	0.3849
	S.E.	0.6246	0.302	0.2173	0.1229	0.1031	0.0916	0.0945	0.0836	0.0758
PB	***Cov***	***0.798***	***0.652***	***0.554***	***0.848***	***0.792***	***0.741***	***0.873***	***0.815***	***0.78***
	Mean	1.1118	0.7169	0.5593	0.5544	0.4733	0.419	0.4747	0.4199	0.3818
	S.E.	0.5049	0.2854	0.1972	0.1193	0.1011	0.0895	0.0923	0.0825	0.0749

As the original sample size n increase, the coverage probability become closer to 0.9 and the mean and standard error of interval lengths are decreasing for each method. It is also observe that when the resample size m increasing, the coverage probability and mean and standard error decreasing.

When the parameter a (scale) increase the coverage probability, the mean and the standard error of the length of the confidence interval decrease. When the parameter b (shape) increasing, the coverage probability increase, but the mean and standard error decreasing.

6. Conclusion

The bootstrap confidence intervals of the process capability index for Weibull distribution have been proposed. The following were considered: The Standard Bootstrap Confidence Interval, Percentile Bootstrap Confidence Interval. Here we want to compare the new Percentile–t bootstrap confidence interval with Standard and Percentile confidence interval and the effect of the Weibull's parameters in performance of bootstrap method. Based on simulation study, The Percentile-t confidence interval provides the highest coverage and shortest mean and standard error of confidence interval length. Although the SB method provide higher coverage probability than PTB method in some cases, but it's mean and standard error of the length of the confidence interval is higher than PTB method. Results show that the PB method provide poorest coverage probability in general.

As a result, when the resample size *m* increase, the coverage probability decrease. It seems that no significant gain is achieved by increasing the resample size. It is better that the resample size be taken equal to the original sample size.

The best coverage probability is provided when the parameter $b \geq 3$, and when the parameter *a* (scale) decrease. In this case the coverage probability become closer to 0.9.

In conclusion, for Weibull distribution with parameters (a,b), Percentile-t bootstrap method has higher coverage probability for all size and it is better to use $b \geq 3$ to access the highest coverage probability. Thus, the standard bootstrap confidence interval is more appropriate than its counterparts in this setting.

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