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# REMEDIAL APPROACHES TO DECREASE THE EFFECT OF MEASUREMENT ERRORS ON SIMPLE LINEAR PROFILE MONITORING

***Abstract:*** *In most profile monitoring applications, the explanatory variables are not fixed from profile to profile. However, in most studies, they are assumed to be fixed values. Furthermore, the observed units usually contain some source of uncertainty referred as "measurement errors." In this paper, first the effect of neglecting the measurement errors effect on detection capability of two common control charts for Phase II monitoring of simple linear profiles is evaluated. Then, three remedial approaches including ranked set sampling, multiple measurement and increasing sample size are utilized to decrease the mentioned effect. Simulation studies in terms of average run length (ARL) metric show that neglecting the measurement errors adversely affects the capability of both charts. The results also confirm that the remedial approaches adequately compensate for the mentioned effect.*

***Keywords:*** *Measurement errors; Multiple measurement approach; Ranked set sampling (RSS); Simple linear profile; Statistical process monitoring (SPM).*

## 1. Introduction

In some production systems, the quality of the process is characterized by a relationship between a response variable and one or more independent explanatory variables. Monitoring such functional relationships over time is referred to as "profile monitoring". Different profile monitoring schemes are classified into two general categories including Phase I and Phase II. The purpose of Phase I monitoring is to provide an analysis on the preliminary data for estimating the model parameters. The main purpose of profile monitoring approaches in Phase II is to design a monitoring scheme for detecting different out-of-control scenarios in the process parameters. The most important application of profile monitoring includes calibration of measurement instruments to ascertain their proper performance over time, determine the optimum calibration frequency, and avoid over-calibration. Furthermore, Kang and Albin (2000) presented a real application where the amount of an artificial sweetener dissolved per liter of water (response variable) is represented by a function of temperature (explanatory variable). Other applications of profile monitoring include agriculture field, optical imaging system, semiconductor manufacturing industry, automotive industry, aluminum electrolytic capacitor manufacturing process, turning process, vertical density of particleboard (please see Maleki et al., 2018).

The most common model in profile monitoring is referred to as "simple linear

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profile" where the behavior of response variable ($y$) depends linearly on the value of a single explanatory variable ($x$). Monitoring simple linear profiles is explored by some researchers such as Kang and Albin (2000), Kim et al. (2003), Mahmoud and Woodall (2004), Saghaei et al. (2009), Noorossana et al. (2011), Yeh and Zerehsaz (2013), Abdella et al. (2014), Kazemzadeh et al. (2016), Khedmati and Niaki (2016). Chiang et al. (2017), Kalaei et al. (2018), Mahmood et al. (2018) and Hassanvand et al. (2019).

Different profile monitoring approaches are generally classified into two major categories: (1) Control charts with fixed explanatory variable(s), and (2) Control charts under random explanatory variable(s). In most researches in the literature such as Zhang et al. (2009), Noorossana et al. (2010), Amiri et al. (2013), Farahani et al. (2014), the $x$ values are considered as fixed values from profile to profile. However, in real world applications, the explanatory variables usually are random quantities. To the best of our knowledge, only few researches such as Noorossana et al. (2015) and Abbas et al. (2019) have taken into account the randomness of explanatory variables.

In statistical process monitoring applications, the samples are taken from the process and plotted for analyzing the process stability and variability. However, because of some inevitable sources of uncertainty, the measured quantities are not completely in accordance with their actual values. This difference between the measured and the actual quantities of products is called as "measurement errors". The effect of measurement errors on the performance of different univariate and multivariate control charts is investigated by several researchers. Examples include Abbasi (2010), Yang et al. (2013), Chakraborty and Khurshid (2013), Hu et al. (2015), Noorossana and Zerehsaz (2015), Hu et al. (2016), Maleki et al. (2016a), Maleki et al. (2016b), Amiri et al. (2018), Salmasnia et al. (2018), Tang et al. (2018), Tran et al. (2019) and Zaidi et al. (2019) and Haq et al. (2020). Readers are referred to the review paper by Maleki et al. (2017) for detailed information.

As noted, in some real statistical process monitoring applications, the process outcome is characterized by profile data instead of univariate or multivariate quality characteristics. A preliminary assumption to construct a control chart to monitor a profile model is that the observed data are accurate and are free from gauge measurement errors. However, exact data is a rare phenomenon in any manufacturing or non-manufacturing environment where human involvement is evident. As far as we know, investigating the effect of measurement errors on the performance of profile monitoring schemes is clearly neglected in the literature. Due to the importance of the issue as well as to fill the mentioned research gap, incorporating a linear covariate error model in constructing two control charts for monitoring simple linear profiles is taken into consideration in this paper. Hence, the first goal of this paper is to study the effect of ignoring the measurement errors on the performance of EWMA-3 and Hotelling $T^2$ charts for monitoring simple linear profiles in the case of random explanatory variable. We provide simulations studies to show how neglecting the measurement errors adversely affect the performance of both charts. As the second goal, we also suggest three remedial approaches for reducing the measurement errors effect on Phase II monitoring of simple linear profiles. The rest of this paper is structured as follows: In section 2, the problem definitions and assumptions are presented. In Section 3, the effect of neglecting measurement errors in constructing EWMA-3 and Hotelling $T^2$ charts under random explanatory variable is discussed. In Section 4, we present simulation studies for evaluating the effect of ignoring measurement errors on the performance of both charts to detect different step shifts in the parameters of simple linear regression model. In Section 5, three remedial approaches including ranked set sampling (RSS), multiple measurement approach as well as increasing sample size are suggested for reducing the effect of measurement errors. In Section 6, the performance of the remedial approaches is investigated via simulation studies. Finally, Section 7 is devoted to conclusion remarks and recommendation for future study.

## 2. Problem definition

As noted, the difference between the measured and the actual quantities caused by the measuring equipment and/or operators is called as measurement errors. In this paper, we focus on Phase II monitoring of simple linear profiles in the case of random explanatory variable which is contaminated by measurement gauge errors. First, the statistical properties of $T^2$ and EWMA-3 charts in terms of *ARL* criterion are investigated when the measurement error is ignored. Then, to lessen the undesired effect of measurement errors, three remedial approaches are proposed. Figure 1 depicts the proposed approach:

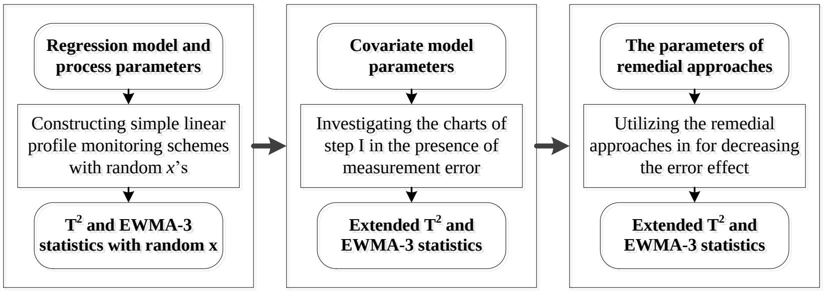


**Figure 1**. The proposed method

The notations and definitions used to formulate the problem are presented in Table 1. According to the mentioned explanations, when the process is in-control, the relationship between the response variable of interest and the random explanatory variable is:

$$y_{ij} = \beta_0 + \beta_1 x_{ij} + \varepsilon_{ij},\ i = 1,\dots,n,\ j = 1,2,\dots \quad (1)$$

where the error term $\varepsilon_{ij}$ and explanatory variable $x_{ij}$ are two independent and normally distributed variables with the following parameters:

$$\varepsilon_{ij} \sim N(0,\sigma_0^2),\ x_{ij} \sim N(\mu_x,\sigma_x^2). \quad (2)$$

The regression parameters in profile $j; j = 1,2,\dots$ can be estimated via ordinary least square (OLS) method according to the following equation:

$$\hat{\beta}_{1j} = \frac{S_{xy(j)}}{S_{xx(j)}},\ \hat{\beta}_{0j} = \bar{y}_j - \hat{\beta}_{1j}\bar{x}_j \quad (3)$$

where

$$S_{xx(j)} = \sum_{i=1}^{n}(x_{ij} - \bar{x}_j)^2, S_{xy(j)} = \sum_{i=1}^{n}(x_{ij} - \bar{x}_j)(y_{ij} - \bar{y}_j) \quad (4)$$

Here the additive error model is:

$$w_{ij} = Ax_{ij} + B + u_{ij}. \quad (5)$$

Typically when $A = 1$ and $B = 0$, we have:

$$w_{ij} = x_{ij} + u_{ij}. \quad (6)$$

It can be concluded that embedding Equation (6) in Equation (1) leads to the following regression model:

$$y_{ij} = \beta_0 + \beta_1(w_{ij} - u_{ij}) + \varepsilon_{ij} = \beta_0 + \beta_1 w_{ij} - \beta_1 u_{ij} + \varepsilon_{ij}; i = 1,2\dots, j = 1,\dots,n \quad (7)$$

Obviously, in the presence of measurement errors, the actual values of explanatory variable are not accessible. Therefore, the estimated values of the regression parameters considering the measurement errors are obtained via the observed explanatory variable as follows:

$$\hat{\beta}_{1j} = \frac{S_{wy(j)}}{S_{ww(j)}},\ \hat{\beta}_{0j} = \bar{y}_j - \hat{\beta}_{1j}\bar{w}_j \qquad (8)$$

where

$$S_{ww(j)} = \sum_{i=1}^{n}(w_{ij} - \bar{w}_j)^2, S_{wy(j)} = \sum_{i=1}^{n}(w_{ij} - \bar{w}_j)(y_{ij} - \bar{y}_j) \qquad (9)$$

**Table 1**. The notations and definitions

| Notation | Description |
|---|---|
| $j$ | Set of profiles |
| $i$ | Set of observations |
| $x_{ij}$ | Observation *i* in profile *j* |
| $\mu_x$ | The mean of explanatory variable |
| $\sigma_x^2$ | The variance of explanatory variable |
| $\varepsilon_{ij}$ | Error term for observation *i* in profile *j* |
| $e_{ij}$ | The residual value for observation $i$ in profile $j$ |
| $\bar{e}_j$ | The average of residuals in profile $j$ |
| $\sigma_0^2$ | In-control variance of error term |
| $\sigma^2$ | Out-of-control variance of error term |
| $u_{ij}$ | measurement error term for observation *i* in profile *j* |
| $\sigma_u^2$ | The variance of measurement error term |
| $w_{ij}$ | The measured quantity of $x_{ij}$ |
| $\beta_0$ | In-control intercept parameter |
| $\beta_1$ | In-control slope parameter |
| $\beta_0'$ | Out-of-control intercept parameter |
| $\beta_1'$ | Out-of-control slope parameter |
| $\hat{\beta}_{0j}$ | The estimated intercept parameter in profile $j$ |
| $\hat{\beta}_{1j}$ | The estimated slope parameter in profile $j$ |
| $E[.]$ | The expected value of the quantity in the brackets |
| $Var[.]$ | The variance of the quantity in the brackets |
| $\mathbf{\Sigma}$ | The variance-covariance matrix |
| $\theta$ | Smoothing parameter of EWMA control chart |
| $n$ | Sample size in each profile |
| $LCL_k$ | Lower control limit of control chart $k$ |
| $UCL_k$ | Upper control limit of control chart $k$ |
| $L_k$ | Control limit coefficient of control chart $k$ |

## 3. Neglecting measurement errors

In this section, two common approaches namely EWMA-3 and Hotelling $T^2$ charts in monitoring simple linear profile under random explanatory variable and gauge measurement errors are discussed.

### 3.1. EWMA-3 approach

One of the most common approaches for monitoring simple linear profiles is EWMA-3 control chart which is first proposed by Kim et al. (2003) in the case of fixed explanatory variable. In EWMA-3 chart, to have independent regression parameters, the *x*-values are transformed so that the average of the transformed explanatory variable in each profile becomes zero (Equation 10).

$$x_{ij}^* = x_{ij} - \bar{x}_j \tag{10}$$

After applying this transformation, the regression model in Equation (1) will be:

$$y_{ij} = \alpha_0 + \alpha_1 x_{ij}^* + \varepsilon_{ij} \quad i = 1,2,...,n, \; j = 1,2,... \tag{11}$$

where

$$\alpha_1 = \beta_1, \; \alpha_0 = \beta_0 + \beta_1 \mu_x. \tag{12}$$

When the random explanatory variable is affected by the measurement errors, it is not possible to directly observe *x*-values. Hence, instead of actual value of explanatory variable, the transformation is performed with respect to measured quantities as follows:

$$w_{ij}^* = w_{ij} - \bar{w}_j \tag{13}$$

After transforming the *w*-values, we can apply three separate charts for monitoring regression parameters including intercept, slope, and standard deviation under measurement errors. The corresponding statistics and control limit for each chart are discussed as follows. The $\text{EWMA}_\text{I}$ statistic (for monitoring intercept parameter) is not affected by the measurement errors (Noorossana and Zerehsaz, 2015). It can be statistically checked that when the explanatory variable has random nature, the variance of response variable ($\text{EWMA}_\text{I}$ statistic) changes from $\sigma_0^2$ to $\sigma_0^2 + \beta_1^2 \sigma_x^2$ ($\frac{\theta}{2-\theta}\frac{\sigma_0^2}{n}$ to $\frac{\theta}{2-\theta}\frac{\sigma_0^2 + \sigma_x^2 \beta_1^2}{n}$). The chart statistic corresponding to *j*th profile: $j = 1,2,...$ for monitoring the intercept parameter will be as:

$$EWMA_I(j) = \theta \hat{\alpha}_{0j} + (1-\theta) EWMA_I(j-1), \tag{14}$$

where $\theta; 0 \le \theta \le 1$ is smoothing parameter and $EWMA_I(0) = \alpha_0 = \beta_0 + \beta_1 \mu_x$. The upper and lower control limits are given by Equations (15) and (16), respectively:

$$\begin{aligned} UCL_I &= \beta_0 + \beta_1 \mu_x + L_I \sqrt{\frac{\theta}{2-\theta}\frac{\sigma_0^2 + \sigma_x^2 \beta_1^2}{n}} \\ &= \alpha_0 + L_I \sqrt{\frac{\theta}{2-\theta}\frac{\sigma_0^2 + \sigma_x^2 \beta_1^2}{n}} \end{aligned} \tag{15}$$

$$\begin{aligned} LCL_I &= \beta_0 + \beta_1 \mu_x - L_I \sqrt{\frac{\theta}{2-\theta}\frac{\sigma_0^2 + \sigma_x^2 \beta_1^2}{n}} \\ &= \alpha_0 - L_I \sqrt{\frac{\theta}{2-\theta}\frac{\sigma_0^2 + \sigma_x^2 \beta_1^2}{n}} \end{aligned} \tag{16}$$

Next, we concentrate on constructing a proper statistic and corresponding control limits for monitoring slope parameter. According to the literature, the control limits for slope parameter, when the values of explanatory variable are error-free and fixed from profile to profile are given as:

$$\begin{aligned} UCL_S &= \beta_1 + L_S \sqrt{\frac{\theta}{2-\theta}\frac{\sigma_0^2}{S_{xx}}} \\ &= \alpha_1 + L_S \sqrt{\frac{\theta}{2-\theta}\frac{\sigma_0^2}{S_{xx}}} \end{aligned} \tag{17}$$

$$\begin{aligned} LCL_S &= \beta_1 - L_S \sqrt{\frac{\theta}{2-\theta}\frac{\sigma_0^2}{S_{xx}}} \\ &= \alpha_1 - L_S \sqrt{\frac{\theta}{2-\theta}\frac{\sigma_0^2}{S_{xx}}} \end{aligned} \tag{18}$$

The control limits in Equations (17) and (18) depend on the value of $S_{xx}$ which is in turn is a function of explanatory variable. Consequently, when the explanatory variable has random nature and is affected by the measurement errors, two problems will be arisen. The first is that the actual values for explanatory variable are not available and the second is regarding to the control limits which will vary from profile to profile. In this case, to have constant control limits, the slope statistic is standardized as follows:

$$Z_{\hat{\alpha}_1}(j) = \frac{\hat{\alpha}_{1j} - \alpha_1}{\sqrt{\dfrac{\sigma_0^2}{S_{ww}}}} = \frac{\hat{\beta}_{1j} - \beta_1}{\sqrt{\dfrac{\sigma_0^2}{S_{ww}}}} \tag{19}$$

The modified EWMA-based slope statistic is obtained as Equation (20):

$$EWMA_S(j) = \theta Z_{\hat{\alpha}_1}(j) + (1-\theta) EWMA_S(j-1), \tag{20}$$

where $EWMA_S(0) = 0$. Now, the modified slope statistic is analyzed with the following constant control limits:

$$UCL_S = L_S \sqrt{\frac{\theta}{2-\theta}} \tag{21}$$

$$LCL_S = -L_S \sqrt{\frac{\theta}{2-\theta}} \tag{22}$$

In the last step of EWMA-3 procedure, the chart statistic for monitoring the error variance is derived as:

$$EWMA_E(j) = \max\{\theta \ln(MSE_j) + (1-\theta) EWMA_E(j-1), \ln(\sigma_0^2)\};\ j = 1,2,... \tag{23}$$

where $EWMA_E(0) = \ln(\sigma_0^2)$ and $MSE_j = \frac{SSE_j}{n-2}$. Note that the value of $SSE_j$ in $j$th profile is calculated by the measured quantities as $SSE_j = \sum_{j=1}^{n} (e_{ij} - \bar{e}_j)^2; e_{ij} = y_{ij} - (\hat{\alpha}_{0j} + \hat{\alpha}_{1j} w_{ij}^*)$.

Since the chart statistic is a positive value, the lower control limit is considered equal to zero. Meanwhile, the upper control limit is obtained based on the following equation:

$$UCL_E = \ln(\sigma_0^2) + L_E \sqrt{\frac{\theta}{2-\theta} \mathrm{var}[\ln(MSE)]} \tag{24}$$

where $Var[\ln(MSE)]$ is obtained via the following formula (Crowder & Hamilton, 1992):

$$Var[\ln(MSE)] = \frac{2}{n-2} + \frac{2}{(n-2)^2} + \frac{4}{3(n-2)^3} - \frac{16}{15(n-2)^5} \tag{25}$$

After deriving the control statistics for each parameter, the designed control scheme signals when at least one of the mentioned statistics falls outside the corresponding control limit interval. The control limits of each control chart are set such that (1) The same in-control average run length ($ARL_0$) value for each method is obtained, (2) The overall $ARL_0$ equals to a desired value.

### 3.2. Hotelling $T^2$

Kang and Albin (2000) proposed Hotelling $T^2$ chart based on the fact that the estimated parameters obtained by least square method are normally distributed. The modified control scheme considering the random explanatory variable under contaminated data by measurement errors is discussed in this subsection. Recall that under measurement errors, the $x$-values are not accessible and instead of the actual values of explanatory variable, the contaminated observations are employed to drive the chart statistic. Despite of EWMA-3 approach, Hotelling $T^2$ control chart uses a single statistic for monitoring model parameters as follows:

$$T_j^2 = (\mathbf{u}_j - \mathbf{u})^T \mathbf{\Sigma}^{-1} (\mathbf{u}_j - \mathbf{u}), \tag{26}$$

where

$$\mathbf{u}_j = [\hat{\beta}_{0j} = \frac{S_{wy(j)}}{S_{ww(j)}}, \hat{\beta}_{1j} = \bar{y}_j - \hat{\beta}_{0j}\bar{w}_j]^T \tag{27}$$

$$\mathbf{u} = [\beta_0, \beta_1]^T \tag{28}$$

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma^2_{\hat{\beta}_0} & \sigma_{\hat{\beta}_0\hat{\beta}_1} \\ \sigma_{\hat{\beta}_0\hat{\beta}_1} & \sigma^2_{\hat{\beta}_1} \end{bmatrix} = \begin{bmatrix} \sigma_0^2(\frac{1}{n} + \frac{\bar{w}^2}{S_{ww}}) & -\frac{\bar{w}\sigma_0^2}{S_{ww}} \\ -\frac{\bar{w}\sigma_0^2}{S_{ww}} & \frac{\sigma_0^2}{S_{ww}} \end{bmatrix} \tag{29}$$

The chart triggers an out-of-control signal when $T_j^2 > UCL_T$ where $UCL_T$ is set such that $ARL_0$ becomes a pre-determined value.

## 4. Simulation studies

In this section the effect of ignoring measurement errors on the performance of EWMA-3 and Hotelling $T^2$ charts is investigated through simulation studies where the relationship between *y* and *x* is expressed by a simple linear model as:

$$y_{ij} = 3 + 2x_{ij} + \varepsilon_{ij}, \tag{30}$$

where $x \sim N(5, \frac{5}{3})$, $\varepsilon \sim N(0,1)$, $n = 4$ and $\theta = 0.2$. We suppose that *x* is affected by the measurement errors according to Equation (6). As mentioned, we use *ARL* values to assess both EWMA-3 and Hotelling $T^2$ control charts in all simulation experiments. *ARL* criterion is defined as the expected number of samples taken from the process until the first sample falls outside the control limits interval. In all simulations, the control limits coefficients of EWMA-3 chart and the *UCL* value of Hotelling $T^2$ chart, $UCL_T$, are set such that we have $ARL_0 = 200$ for both methods. To do that, $L_I, L_S$ and $L_E$ are chosen separately such that for each of EWMA-3 charts, the $ARL_0$ becomes approximately equal to 599. This leads to obtain the overall $ARL_0 = 200$. The control limits coefficients corresponding to each control chart for EWMA-3 are reported in the first rows of each Table. The effect of ignoring measurement errors on the performance of EWMA-3 chart to detect shifts in $\beta_0$, $\beta_1$ and $\sigma$ under different values of $\sigma_u^2$ are given in Tables 2-4, respectively. The results of Tables 2-4 show that ignoring the measurement errors can adversely affect the ability of EWMA-3 control chart in detecting all regression model parameters. It is represented that the *ARL*s increase as the value of $\sigma_u^2$ increases.

**Table 2**. *ARLs* of EWMA-3 control chart for different values of $\sigma_u^2$ under shifts in $\beta_0$

| $L_I$ | 3.0156 | 3.0276 | 3.0289 | 3.0294 |
|---|---|---|---|---|
| $L_S$ | 3.0109 | 3.0869 | 3.3521 | 3.8881 |
| $L_E$ | 1.3723 | 1.4374 | 1.6094 | 1.8575 |
| $\beta_0'$ \ $\sigma_u^2$ | No Error | 0.01 | 0.04 | 0.09 |
| 3 | 199.6620 | 200.3130 | 199.9450 | 201.7490 |
| 3.2 | 151.7380 | 155.7440 | 165.0300 | 168.1360 |
| 3.4 | 91.4240 | 96.4000 | 98.8860 | 100.8840 |
| 3.6 | 51.0290 | 53.7810 | 55.8960 | 56.3800 |
| 3.8 | 29.8120 | 30.7770 | 32.1410 | 32.4030 |
| 4 | 19.2230 | 19.8770 | 20.3880 | 21.0420 |
| 4.2 | 13.7840 | 14.5110 | 14.6610 | 14.9760 |
| 4.4 | 10.3120 | 10.8540 | 10.9880 | 11.2330 |
| 4.6 | 8.2520 | 8.4090 | 8.6880 | 8.7770 |
| 4.8 | 6.9030 | 6.9470 | 7.1670 | 7.2650 |
| 5 | 5.8020 | 6.0280 | 6.1150 | 6.1620 |

**Table 3**. *ARL*s of EWMA-3 control chart for different values of $\sigma_u^2$ under shifts in $\beta_1$

| $L_I$ | 3.0156 | 3.0276 | 3.0289 | 3.0294 |
|---|---|---|---|---|
| $L_S$ | 3.0109 | 3.0869 | 3.3521 | 3.8881 |
| $L_E$ | 1.3723 | 1.4374 | 1.6094 | 1.8575 |
| $\beta_1'$ \ $\sigma_u^2$ | No Error | 0.01 | 0.04 | 0.09 |
| 2 | 199.6620 | 200.3130 | 199.9450 | 201.7490 |
| 2.025 | 163.1000 | 188.5590 | 190.7970 | 194.2240 |
| 2.050 | 119.5190 | 128.3730 | 145.4930 | 149.5680 |
| 2.075 | 78.8010 | 88.0480 | 95.3280 | 102.4580 |
| 2.100 | 51.2480 | 58.7590 | 64.6000 | 65.2050 |
| 2.125 | 34.7670 | 38.6940 | 43.8960 | 44.4290 |
| 2.150 | 25.9270 | 26.5180 | 31.8310 | 32.2570 |
| 2.175 | 19.2660 | 20.9920 | 23.8190 | 24.6760 |
| 2.200 | 15.6860 | 17.3510 | 18.0290 | 18.5820 |
| 2.225 | 12.9160 | 13.0930 | 14.5630 | 15.4390 |
| 2.250 | 10.7300 | 11.1300 | 12.0700 | 12.4890 |

**Table 4**. *ARL*s of EWMA-3 control chart for different values of $\sigma_u^2$ under shifts in $\sigma$

| $L_I$ | 3.0156 | 3.0276 | 3.0289 | 3.0294 |
|---|---|---|---|---|
| $L_S$ | 3.0109 | 3.0869 | 3.3521 | 3.8881 |
| $L_E$ | 1.3723 | 1.4374 | 1.6094 | 1.8575 |
| $\sigma$ \ $\sigma_u^2$ | No Error | 0.01 | 0.04 | 0.09 |
| 1 | 199.6620 | 200.3130 | 199.9450 | 201.7490 |
| 1.2 | 37.0520 | 41.1810 | 49.9270 | 56.3350 |
| 1.4 | 14.0850 | 15.0720 | 18.9240 | 21.6090 |
| 1.6 | 7.7170 | 8.2060 | 10.1920 | 12.1700 |
| 1.8 | 5.5040 | 5.9030 | 7.0490 | 8.5390 |
| 2 | 4.2910 | 4.4200 | 5.4390 | 6.3870 |
| 2.2 | 3.5250 | 3.7400 | 4.3960 | 5.0290 |
| 2.4 | 3.0250 | 3.322 | 3.7860 | 4.3620 |
| 2.6 | 2.7410 | 2.8640 | 3.3540 | 3.7460 |
| 2.8 | 2.4270 | 2.6160 | 2.9760 | 3.4480 |
| 3 | 2.2010 | 2.4030 | 2.7410 | 3.1750 |

Tables 5-7 contains *ARL*s of Hotelling $T^2$ control chart in detecting model parameters under covariate model presented in Equation (6) and different values of $\sigma_u^2$. Similar to EWMA-3 control chart, The *ARL*s reported in Tables 5-7 confirm that the ability of Hotelling $T^2$ control chart in detecting all regression model parameters is affected by the measurement errors. The results also reveal that as $\sigma_u^2$ increases, the *ARL*s tend to increase. By comparing the results of Tables 2-4 with those given in Tables 5-7, we can see that in the presence of measurement errors which are ignored, the Hotelling $T^2$ control chart performs better than EWMA-3 chart.

**Table 5**. *ARL*s of Hotelling $T^2$ control chart for different values of $\sigma_u^2$ under shifts in $\beta_0$

| $UCL_T$ | 10.5966 | 11.0966 | 12.2966 | 14.6166 |
|---|---|---|---|---|
| $\sigma_u^2$ / $\beta_0'$ | No Error | 0.01 | 0.04 | 0.09 |
| 3 | 200.2980 | 199.0770 | 199.8970 | 199.1050 |
| 3.2 | 138.6150 | 143.2200 | 148.8170 | 150.8100 |
| 3.4 | 63.3120 | 67.0110 | 71.6190 | 83.9860 |
| 3.6 | 27.4160 | 28.9130 | 33.8550 | 38.5910 |
| 3.8 | 12.6390 | 13.6130 | 17.4020 | 19.2940 |
| 4 | 6.7820 | 7.4510 | 8.6000 | 11.1720 |
| 4.2 | 3.8730 | 4.3050 | 4.9670 | 6.5910 |
| 4.4 | 2.5630 | 2.7650 | 3.2840 | 4.0320 |
| 4.6 | 1.8660 | 1.9520 | 2.3170 | 2.7500 |
| 4.8 | 1.4260 | 1.5200 | 1.7740 | 2.0140 |
| 5 | 1.2040 | 1.2450 | 1.4230 | 1.5840 |

**Table 6**. *ARL*s of Hotelling $T^2$ control chart for different values of $\sigma_u^2$ under shifts in $\beta_1$

| $UCL_T$ | 10.5966 | 11.0966 | 12.2966 | 14.6166 |
|---|---|---|---|---|
| $\sigma_u^2$ / $\beta_1'$ | No Error | 0.01 | 0.04 | 0.09 |
| 2 | 200.2980 | 199.0770 | 199.8970 | 199.1050 |
| 2.025 | 167.2550 | 173.1380 | 173.2770 | 183.8580 |
| 2.050 | 106.4040 | 112.7970 | 124.4000 | 130.0830 |
| 2.075 | 64.7410 | 68.6230 | 77.7720 | 87.7420 |
| 2.100 | 37.5110 | 39.9930 | 49.4520 | 53.7320 |
| 2.125 | 22.1240 | 25.1840 | 29.7610 | 35.8630 |
| 2.150 | 13.9380 | 14.8820 | 19.2350 | 23.3280 |
| 2.175 | 9.2940 | 10.0300 | 11.6400 | 15.5490 |
| 2.200 | 6.0780 | 6.4000 | 8.1150 | 10.5910 |
| 2.225 | 4.1940 | 4.5750 | 5.9970 | 7.2420 |
| 2.250 | 3.2950 | 3.4380 | 4.1920 | 5.1450 |

**Table 7.** *ARL*s of Hotelling $T^2$ control chart for different values of $\sigma_u^2$ under shifts in $\sigma$

| $UCL_T$ | 10.5966 | 11.0966 | 12.2966 | 14.6166 |
|---|---|---|---|---|
| $\sigma_u^2$ / $\sigma$ | No Error | 0.01 | 0.04 | 0.09 |
| 1 | 200.2980 | 199.0770 | 199.8970 | 199.1050 |
| 1.2 | 40.1760 | 40.2870 | 47.2910 | 54.3630 |
| 1.4 | 14.5130 | 15.6660 | 18.8220 | 22.3790 |
| 1.6 | 7.9980 | 8.2380 | 9.7950 | 11.6030 |
| 1.8 | 4.9050 | 5.4210 | 6.1650 | 7.3340 |
| 2 | 3.7960 | 3.8980 | 4.5820 | 5.4540 |
| 2.2 | 3.0370 | 3.2600 | 3.6470 | 3.9170 |
| 2.4 | 2.5540 | 2.6250 | 2.7960 | 3.3570 |
| 2.6 | 2.1620 | 2.2130 | 2.5060 | 2.6950 |
| 2.8 | 1.9670 | 2.0580 | 2.2380 | 2.4240 |
| 3 | 1.8110 | 1.8560 | 1.9520 | 2.3130 |

## 5. Remedial approaches

In the previous sections, we proved the undesired effect of ignoring the measurement errors on monitoring simple linear profiles. To consider such errors, the statistics and the corresponding control limits should be modified as follows (see also Fuller, 1987). As measurement errors have no effect on the EWMA$_I$ chart, no modification is required. However, The EWMA$_S$ and EWMA$_E$ statistics and their corresponding control limits should be modified. In this regard, the standardized slope statistic is given as:

$$Z'_{\hat{\alpha}_1}(j)=\frac{\hat{\alpha}_{1j}-\lambda\alpha_1}{\sqrt{\frac{\sigma_y^2-\sigma_{wy}\lambda\alpha_1}{S_{ww}}}}=\frac{\hat{\beta}_{1j}-\lambda\beta_1}{\sqrt{\frac{\sigma_y^2-\sigma_{wy}\lambda\beta_1}{S_{ww}}}} \quad (31)$$

where $\lambda=\frac{\sigma_x^2}{\sigma_x^2+\sigma_u^2}$ is the reliability ratio, $\sigma_y^2=\sigma_0^2+\beta_1^2\sigma_x^2$ and . $\sigma_{wy}=\beta_1\sigma_x^2$ Then:

$$EWMA_S(j)=\theta Z'_{\hat{\alpha}_1}(j)+(1-\theta)EWMA_S(j-1) \quad (32)$$

where $EWMA_S(0)=0$ and the control limits are computed via Equations (21) and (22). Next, The EWMA$_E$ statistic in the presence of measurement errors is given by:

$$EWMA_E(j)=\max\{\theta[MSE_j-(\sigma_0^2+\lambda\sigma_u^2\beta_1)]+(1-\theta)EWMA_E(j-1),0\};\ j=1,2,... \quad (33)$$

where $EWMA_E(0)=0$ .Then:

$$UCL_E=L_E\sqrt{\frac{2(\sigma_0^2+\lambda\sigma_u^2\beta_1)^2}{n-2}\frac{\theta}{2-\theta}} \quad (34)$$

The $T^2$ statistic for $j$th profile in the presence of measurement errors is similar to Equation (26), where:

$$\mathbf{u}=[\beta_0+\beta_1\mu_x(1-\lambda),\lambda\beta_1]^T \quad (35)$$

and

$$\mathbf{\Sigma}=\begin{bmatrix}\sigma^2_{\hat{\beta}_0} & \sigma_{\hat{\beta}_0\hat{\beta}_1}\\ \sigma_{\hat{\beta}_0\hat{\beta}_1} & \sigma^2_{\hat{\beta}_1}\end{bmatrix}=$$

$$\begin{bmatrix}\frac{(\sigma_0^2+\lambda\sigma_u^2\beta_1)}{n}+\bar{w}^2\frac{(\sigma_y^2-\sigma_{wy}\lambda\beta_1)}{S_{ww}} & -\bar{w}\frac{(\sigma_y^2-\sigma_{wy}\lambda\beta_1)}{S_{ww}}\\ -\bar{w}\frac{(\sigma_y^2-\sigma_{wy}\lambda\beta_1)}{S_{ww}} & \frac{(\sigma_y^2-\sigma_{wy}\lambda\beta_1)}{S_{ww}}\end{bmatrix} \quad (36)$$

Now to reduce the adverse effect of measurement errors, three remedial approaches are developed in the following sub-sections.

### 5.1. Ranked set sampling spproach

In this subsection, first the RSS method is briefly explained. Then, utilizing this strategy for the sake of reducing the effect of measurement error is given. Let $X$ follows a given distribution with mean $\mu_x$ and variance $\sigma_x^2$ whose probability density function and cumulative distribution function are denoted by $f(x)$ and $F(x)$, respectively. Suppose that vector $\mathbf{X}=(x_1,x_2,...,x_n)^T$ denote a simple random sample of size $n$ taken from $f(x)$ while $\mathbf{X'}=(x_{(1:n)},x_{(2:n)},...,x_{(n:n)})^T$ is the ordered statistics of the corresponding sample. The mean and variance of $i$th; $i=1,2,...,n$ ordered statistic can be obtained by Equations (41) and (42):

$$\mu_{x(i:m)}=\int xf_{(i:m)}(x)dx \quad (37)$$

$$\sigma^2_{x(i:m)}=\int(x-\mu_{x(i:m)})^2 f_{(i:m)}(x)dx \quad (38)$$

where $f_{(i:m)}(x)$ is the probability density function of $x_{(i:m)}$ which is obtained by Equation (39):

$$f_{(i:m)}(x)=\frac{m!}{(i-1)!(m-i)!}\{F(x)\}^{i-1}\{1-F(x)\}^{m-i}f(x),\ -\infty<x<\infty \quad (39)$$

For more information, the readers can also see David and Nagaraja (2003). Now the utilization of RSS strategy in our research is discussed. Let $\mathbf{W_j} = (w_{1j}, w_{2j}, ..., w_{1n})^T$ be a simple random sample of size *n* from the measured quantities. In order to incorporate the RSS strategy in simple linear profile monitoring approaches under conatminated *x*-values, the following steps are recommended:

1- Take *n* samples (profile) each of size *n*: Suppose that *n* simple random samples in each sampling cycle are $\mathbf{W_j}; j = 1, 2, ..., n$ .

2- Sort the measured quantities within each profile: After sorting the quantities, the sample vector for jth; sample is denoted by $\mathbf{W'_j} = (w_{(1:n)j}, w_{(2:n)j}, ..., w_{(n:n)n})^T$ .

3- The smallest ranked unit is selected from the first profile and the second smallest ranked unit is selected from the second set. The procedure continues and the largest ranked unit is selected from the last set.

4- This completes one cycle of a ranked set sample of size *n*.

5- $S_{ww(j)}$ and $S_{wy(j)}$ in Equation (8), are computed based on the ranked set samples instead of the simple random samples.

Note that the transformation in EWMA-3 must be performed in respect to the ranked set sampling of the measure quantities.

### 5.2. Multiple measurement approach

One of the most common approaches for covering the measurement error is taking several measurements on each sample point which is also applied by Linna and Woodall (2001) as well as Costa and Castagliola (2011). Taking multiple measurements at each observation of the underlying quality characteristic generally leads to a smaller variance of the error component (Haq et al., 2015). In this section, taking multiple measurement on each observation of a given profile is discussed. Let $y_{ij1}, y_{ij2}, ..., y_{ijk}$ be *k* measurements which are taken for $y_{ij}; i = 1, ..., n, j = 1, 2, ...$. Considering the covariate model in Equation (6), the variance of the measured quantity will be:

$$\sigma_w^2 = \sigma_x^2 + \sigma_u^2 \tag{40}$$

As the variance of the error term in Equation (40) increases the difference between the actual and measured quality characteristic will increase. It can be statistically checked that by taking *k* measurements on each sample, the variance of the measured quality characteristic reduces to $\sigma_w^2 = \sigma_x^2 + \frac{\sigma_u^2}{k}$ .

Obviously by utilizing this approach, the difference between the actual and measured quantities decrease and consequently more reliable measurements are obtained.

### 5.3. Incerasing sample size

It is proved in statistical process monitoring literature that as the sample size increases, the power of a control scheme in detecting process faults increases (Montgomery, 2005). This issue confirms that increasing the sample size results in decreasing of the variance of statistic. Consequently, it can be employed as third remedial approach to reduce the undesirable effect of measurement errors in monitoring a simple linear profile.

## 6. Performance evaluation of remedial approaches

The performance of the proposed remedial approaches to reduce the measurement errors effect is analyzed and compared in this section. For this purpose, the example presented in section 4 is also used in this section. The results of utilizing the remedial approach in EWMA-3 and Hotelling $T^2$ control charts are reported in Tables 8-10 and 11-13, respectively. The results given in Tables 8-10 represent that both RSS and multiple measurements approaches compensate for the undesirable effects of measurement errors on detecting ability of EWMA-3 control chart. However, the RSS method outperforms the multiple measurement approach under step shifts in $\beta_0$ and $\beta_1$. We can see that when $k=5$, the *ARL*s are close to those obtained under no error scenario. It could be observed from Tables 8-10 that increasing the sample size effectively improves the detecting ability of EWMA-3 control chart in detecting all out-of-control scenarios.

Tables 11-13 represent the results of utilizing remedial approaches in Hotelling $T^2$ control chart for monitoring intercept parameter, slope parameter and error variance, respectively. It can be seen from Tables 11-13 that in multiple measurement approach, using $k=5$ measurements per observation can adequately cover the measurement errors effect. Similar to EWMA-3 control chart, both RSS and increasing sample size approaches can also reduce the *ARL*s in all out-of-control scenarios. However, the performance of increasing the sample size in improving the detecting ability of Hotelling $T^2$ control chart is more considerable.

**Table 8**. *ARL*s of remedial methods in EWMA-3 chart under shifts in $\beta_0$ when $\sigma_u^2 = 0.04$

| $L_I$ | 3.0290 | 3.0201 | 3.0189 | 3.0172 | 2.1676 | 3.0290 | 3.0290 | 3.0290 |
|---|---|---|---|---|---|---|---|---|
| $L_S$ | 3.0241 | 3.0204 | 3.0114 | 3.0074 | 3.0269 | 3.0241 | 3.0241 | 3.0241 |
| $L_E$ | 4.5584 | 4.2984 | 4.2524 | 4.2134 | 4.5494 | 4.2684 | 4.1468 | 4.0826 |
| $\beta_0'$ | *k*=1 | Multiple measurement | | | RSS | increasing sample size | | |
| | | *k*=2 | *k*=3 | *k*=5 | | *n*=6 | *n*=8 | *n*=10 |
| 3 | 199.643 | 200.816 | 199.534 | 199.522 | 200.253 | 201.78 | 199.204 | 199.741 |
| 3.2 | 160.797 | 155.92 | 153.913 | 153.824 | 137.183 | 149.896 | 129.361 | 123.387 |
| 3.4 | 97.885 | 96.537 | 93.781 | 92.312 | 61.138 | 73.684 | 60.657 | 46.323 |
| 3.6 | 54.554 | 54.054 | 51.162 | 51.048 | 29.063 | 38.613 | 27.923 | 23.545 |
| 3.8 | 31.866 | 31.769 | 31.023 | 30.691 | 15.911 | 20.702 | 15.744 | 13.340 |
| 4 | 20.734 | 20.394 | 20.147 | 19.876 | 10.589 | 12.859 | 10.422 | 8.643 |
| 4.2 | 14.564 | 14.499 | 14.401 | 13.824 | 7.972 | 9.417 | 7.809 | 6.438 |
| 4.4 | 10.912 | 10.820 | 10.538 | 10.334 | 6.161 | 7.597 | 6.145 | 5.265 |
| 4.6 | 8.646 | 8.626 | 8.498 | 8.48 | 5.102 | 6.227 | 5.030 | 4.337 |
| 4.8 | 7.125 | 6.970 | 6.956 | 6.925 | 4.356 | 5.206 | 4.381 | 3.798 |
| 5 | 5.988 | 5.945 | 5.932 | 5.867 | 3.785 | 4.557 | 3.806 | 3.228 |

**Table 9**. *ARL*s of remedial methods in EWMA-3 chart under shifts in $\beta_1$ when $\sigma_u^2 = 0.04$

| $L_I$ | 3.0289 | 3.0201 | 3.0189 | 3.0172 | 2.1676 | 3.0290 | 3.0290 | 3.0290 |
|---|---|---|---|---|---|---|---|---|
| $L_S$ | 3.0241 | 3.0204 | 3.0114 | 3.0074 | 3.0269 | 3.0241 | 3.0241 | 3.0241 |
| $L_E$ | 4.5584 | 4.2984 | 4.2524 | 4.2134 | 4.5494 | 4.2684 | 4.1468 | 4.0826 |
| $\beta_1'$ | *k*=1 | Multiple measurement | | | RSS | increasing sample size | | |
| | | *k*=2 | *k*=3 | *k*=5 | | *n*=6 | *n*=8 | *n*=10 |
| 2 | 199.643 | 200.816 | 199.534 | 199.522 | 200.253 | 201.78 | 199.204 | 199.741 |
| 2.025 | 174.888 | 171.951 | 168.987 | 162.335 | 165.086 | 168.331 | 151.158 | 144.815 |
| 2.050 | 124.286 | 121.666 | 119.178 | 112.461 | 99.890 | 99.066 | 85.288 | 78.703 |
| 2.075 | 82.772 | 81.381 | 81.345 | 81.307 | 58.908 | 63.828 | 46.214 | 41.233 |
| 2.100 | 55.343 | 54.097 | 53.776 | 53.037 | 34.670 | 39.679 | 31.015 | 26.025 |
| 2.125 | 38.435 | 37.321 | 36.696 | 35.970 | 22.220 | 26.346 | 20.492 | 16.554 |
| 2.150 | 27.820 | 27.320 | 27.193 | 26.926 | 16.634 | 18.927 | 14.796 | 12.159 |
| 2.175 | 22.265 | 20.303 | 20.079 | 19.732 | 12.236 | 14.438 | 11.364 | 9.383 |
| 2.200 | 17.147 | 15.998 | 15.947 | 15.547 | 10.052 | 11.696 | 9.240 | 7.586 |
| 2.225 | 14.319 | 13.394 | 13.243 | 13.169 | 8.144 | 9.813 | 7.325 | 6.359 |
| 2.250 | 11.171 | 10.946 | 10.754 | 10.729 | 6.824 | 7.886 | 6.454 | 5.674 |

**Table 10**. *ARL*s of remedial methods in EWMA-3 chart under shifts in $\sigma$ when $\sigma_u^2 = 0.04$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $L_I$ | 3.0290 | 3.0201 | 3.0189 | 3.0172 | 2.1676 | 3.0290 | 3.0290 | 3.0290 |
| $L_S$ | 3.0241 | 3.0204 | 3.0114 | 3.0074 | 3.0269 | 3.0241 | 3.0241 | 3.0241 |
| $L_E$ | 4.5584 | 4.2984 | 4.2524 | 4.2134 | 4.5494 | 4.2684 | 4.1468 | 4.0826 |
| $\sigma$ | *k*=1 | Multiple measurement | | | RSS | increasing sample size | | |
| | | *k*=2 | *k*=3 | *k*=5 | | *n*=6 | *n*=8 | *n*=10 |
| 1 | 199.643 | 200.816 | 199.534 | 199.522 | 200.253 | 201.78 | 199.204 | 199.741 |
| 1.2 | 45.528 | 40.580 | 38.331 | 37.696 | 43.160 | 29.313 | 22.873 | 17.319 |
| 1.4 | 16.130 | 14.491 | 13.663 | 13.474 | 15.798 | 9.869 | 7.483 | 6.036 |
| 1.6 | 8.885 | 7.836 | 7.659 | 7.625 | 8.638 | 5.441 | 4.183 | 3.551 |
| 1.8 | 5.873 | 5.156 | 5.001 | 4.893 | 5.629 | 3.795 | 2.963 | 2.516 |
| 2 | 4.479 | 3.983 | 3.890 | 3.814 | 4.242 | 2.754 | 2.303 | 1.956 |
| 2.2 | 3.607 | 3.194 | 3.123 | 3.078 | 3.448 | 2.399 | 1.968 | 1.714 |
| 2.4 | 3.051 | 2.745 | 2.645 | 2.635 | 2.871 | 2.021 | 1.669 | 1.487 |
| 2.6 | 2.456 | 2.440 | 2.337 | 2.313 | 2.481 | 1.789 | 1.486 | 1.332 |
| 2.8 | 2.277 | 2.083 | 2.067 | 2.058 | 2.255 | 1.583 | 1.371 | 1.227 |
| 3 | 2.020 | 1.913 | 1.897 | 1.862 | 2.052 | 1.473 | 1.263 | 1.14 |

**Table 11.** *ARL*s of remedial approaches in $T^2$ chart under shifts in $\beta_0$ when $\sigma_u^2 = 0.04$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *UCL* | 15.6866 | 15.5729 | 15.5599 | 10.5178 | 15.4896 | 12.0000 | 10.4603 | 9.7187 |
| $\beta_0'$ | *k*=1 | Multiple measurement | | | RSS | Increasing sample size | | |
| | | *k*=2 | *k*=3 | *k*=5 | | *n*=6 | *n*=8 | *n*=10 |
| 3 | 202.2075 | 199.4289 | 200.2399 | 202.3810 | 202.6230 | 201.0561 | 199.7367 | 200.0940 |
| 3.2 | 131.5736 | 126.3640 | 121.1526 | 120.2241 | 129.2083 | 118.5529 | 116.2991 | 123.0383 |
| 3.4 | 55.4392 | 52.9146 | 51.6215 | 50.1912 | 55.9009 | 44.6357 | 42.4279 | 44.6097 |
| 3.6 | 24.5755 | 23.1632 | 22.1306 | 21.6579 | 24.8869 | 17.9641 | 15.6114 | 15.6941 |
| 3.8 | 12.1850 | 11.0994 | 10.8567 | 10.3899 | 12.0005 | 8.0928 | 6.8284 | 6.3067 |
| 4 | 6.5763 | 5.9643 | 5.8161 | 5.6716 | 6.4905 | 4.2584 | 3.4097 | 3.0243 |
| 4.2 | 4.0398 | 3.5887 | 3.4584 | 3.3865 | 3.9495 | 2.5607 | 2.0834 | 1.8430 |
| 4.4 | 2.6240 | 2.4267 | 2.3764 | 2.2856 | 2.6177 | 1.7758 | 1.4835 | 1.3433 |
| 4.6 | 1.8929 | 1.7569 | 1.7464 | 1.7052 | 1.8806 | 1.4115 | 1.1988 | 1.1144 |
| 4.8 | 1.4988 | 1.4182 | 1.4017 | 1.3805 | 1.4866 | 1.1873 | 1.0759 | 1.0390 |
| 5 | 1.2796 | 1.2279 | 1.2119 | 1.1879 | 1.2657 | 1.0792 | 1.0247 | 1.0099 |

**Table 12**. *ARL*s of remedial approaches in $T^2$ chart under shifts in $\beta_1$ when $\sigma_u^2 = 0.04$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *UCL* | 15.6866 | 15.5729 | 15.5599 | 10.5178 | 15.4896 | 12.0000 | 10.4603 | 9.7187 |
| $\beta_1'$ | *k*=1 | Multiple measurement | | | RSS | Increasing sample size | | |
| | | *k*=2 | *k*=3 | *k*=5 | | *n*=6 | *n*=8 | *n*=10 |
| 2 | 202.2075 | 199.4289 | 200.2399 | 202.3810 | 202.6230 | 201.0561 | 199.7367 | 200.0940 |
| 2.025 | 170.9360 | 167.3911 | 160.6785 | 163.7788 | 169.5597 | 157.1161 | 164.1008 | 171.6316 |
| 2.050 | 107.1641 | 105.1903 | 106.3814 | 104.6275 | 104.4821 | 96.0579 | 97.5662 | 100.1347 |
| 2.075 | 63.8460 | 63.0219 | 62.0686 | 61.6160 | 63.9408 | 53.4925 | 51.4281 | 55.5351 |
| 2.100 | 38.7724 | 36.5657 | 35.6803 | 34.9352 | 35.8472 | 29.5911 | 26.9286 | 28.3027 |
| 2.125 | 22.5777 | 21.5263 | 21.4652 | 21.6798 | 23.1531 | 16.8320 | 14.5689 | 14.5148 |
| 2.150 | 15.1654 | 13.4507 | 13.5525 | 13.1943 | 14.1029 | 10.2177 | 8.6438 | 7.6737 |
| 2.175 | 9.7245 | 8.8835 | 8.7131 | 8.6377 | 9.6917 | 6.3543 | 5.0994 | 4.5853 |
| 2.200 | 6.6023 | 5.9712 | 5.9076 | 5.6589 | 6.2834 | 4.1534 | 3.4372 | 3..0048 |
| 2.225 | 4.6684 | 4.3342 | 4.2180 | 4.1512 | 4.6617 | 3.0729 | 2.4599 | 2.1406 |
| 2.250 | 3.4195 | 3.1705 | 3.2157 | 3.1072 | 3.4144 | 2.2580 | 1.8480 | 1.6372 |

**Table 13.** *ARL*s of remedial approaches in $T^2$ chart under shifts in $\sigma$ when $\sigma_u^2 = 0.04$

| *UCL* | 15.6866 | 15.5729 | 15.5599 | 10.5178 | 15.4896 | 12.0000 | 10.4603 | 9.7187 |
|---|---|---|---|---|---|---|---|---|
| $\sigma$ | *k*=1 | Multiple measurement | | | RSS | increasing sample size | | |
| | | *k*=2 | *k*=3 | *k*=5 | | *n*=6 | *n*=8 | *n*=10 |
| 1 | 202.2075 | 199.4289 | 200.2399 | 202.3810 | 202.6230 | 201.0561 | 199.7367 | 200.0940 |
| 1.2 | 101.6049 | 94.7838 | 92.2298 | 94.1701 | 99.6060 | 95.5580 | 95.1960 | 94.6360 |
| 1.4 | 56.2428 | 52.3460 | 50.9836 | 51.7580 | 57.7461 | 51.8065 | 50.7440 | 53.9060 |
| 1.6 | 37.3530 | 34.3042 | 33.0529 | 32.6370 | 37.4673 | 33.4905 | 34.2365 | 35.2810 |
| 1.8 | 27.6202 | 24.3028 | 24.8623 | 23.2745 | 26.1062 | 24.4775 | 23.9700 | 25.5295 |
| 2 | 19.2646 | 18.4189 | 17.4450 | 17.4702 | 19.6970 | 18.0755 | 18.1560 | 18.7440 |
| 2.2 | 16.0198 | 14.5617 | 13.9818 | 13.3626 | 15.8358 | 13.9950 | 14.1615 | 14.8410 |
| 2.4 | 12.6644 | 12.0286 | 11.5582 | 11.4561 | 12.3729 | 11.4600 | 11.4495 | 11.7190 |
| 2.6 | 10.9065 | 9.7943 | 9.7419 | 9.2139 | 10.4800 | 9.4940 | 9.7210 | 10.1985 |
| 2.8 | 9.3397 | 8.2912 | 8.0791 | 8.0429 | 8.8343 | 8.2095 | 8.3885 | 8.5355 |
| 3 | 8.1601 | 7.3439 | 7.1449 | 7.1281 | 7.9894 | 7.5210 | 7.2140 | 7.4815 |

## 7. Conclusion and future study

As a relatively new area in the context of SPM, profile monitoring has gained much attentions after the review paper by Woodall (2007). To the best of the authors' knowledge, all profile monitoring approaches have neglected the effect of errors cause by measuring instrument, environment and work-pieces on the capability of control charts. To address this issue, the effect of ignoring measurement errors on the performance of EWMA-3 and Hotelling $T^2$ charts to monitor simple liner profiles under random explanatory variable was investigated in this paper. Thorough simulation studies, we showed that ignoring the measurement errors adversely affects the detecting ability of the both mentioned charts. We also showed that as the variance of measurement error term increases, the ability of both charts to detect step shifts in intercept parameter, slope parameter and error variance reduces. In order to compensate the measurement errors effects, three remedial approaches including ranked set sampling, multiple measurements per unit as well as increasing sample size were utilized. The results showed that all suggested remedial approaches can adequately reduce the effect of measurement errors. The methods discussed in this paper have been proposed under three general limitations. First we considered a simple linear regression model to express the relationship between response and explanatory variables. Second, it was assumed that the response variable follows a Normal distribution. The last limitation is the assumption in which the observations within each profile are independent from each other. Concerning the first and second limitations, considering other types of regression models under non-normality assumption such as generalized linear models with Binary or Poisson response data is recommended as the future research. Developing time-series models to address autocorrelation structure of data is suggested to fulfill the third limitation. Moreover, utilizing other remedial approaches for decreasing the errors effect on profiling monitoring control charts could be mentioned as another research direction

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