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# DETERMINATION OF CRITICAL ROTATIONAL SPEED OF CIRCULAR SAWS FROM NATURAL FREQUENCIES OF ANNULAR PLATE WITH ANALOGOUS DIMENSIONS

***Abstract:*** *It is suitable to reduce thickness of circular saw when trying to enhance usability of wood raw material, but reducing thickness also causes reduction of permissible rotational speed which reduces sawing speed. If one increase circular saw rotational speed over permissible one the quality of machined surfaces will reduce because of enhanced vibrations. Permissible rotational speed can be calculated from critical rotational speed which can be defined from natural frequencies of the saw. In this article critical rotational speeds of standard clamped saws (with flat disk surface and without slots) are calculated by using finite element method and classical theory of thin plates on annular plates. Mode shapes and natural frequencies of annular plates are determined by using Bessel functions and by using polynomial functions. Obtained results suggest that standard clamped circular saws without slots and with relatively small teeth can be determined from classical theory of thin plates for annular plates with accuracy depending on clamping ratio.*

***Keywords:*** *circular saw, theory of thin plates, annular plates, critical rotational speed*

## 1. Introduction

Circular saw manufacturers tends to reduce thickness of saws and to enhance their rotational speeds with the aim of increasing the usability of wood raw material or other types of materials (Ucun, 2012; Fragassa *et al.*, 2016; Fragassa *et al.*, 2016). But, circular saw blade thickness has to satisfy needed lateral stiffness (Stakhiev, 2000) for appropriate sawing workload and appropriate working circular speed which won't cause high thermal load (Anđelić *et al.*, 2016) and decreasing of lateral stiffness. Also, reducing of circular saw thickness can cause occurrence of high vibrations which appears if working circular speed is higher than permissible circular speed which decreases quality of processing (ie. increases roughness and decreases accuracy of workpiece dimensions), enhance noise and decrease tool durability. Also, non-linear vibrations can be self-excited at circular saws (Raman and Mote, 1999) or bandsaws (Žigulić *et al.*, 2015). If increased vibrations happen the solution is to decrease the speed of sawing and the workpiece feed speed (Angelo and Mote, 1988) which leads to

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lower efficiency. Specified reasons limit thinning of circular saw and it is needed to find an optimum.

Permissible rotational speed is determined from critical rotational speed which is the maximum rotational speed when circular saw rotate with standardized stability (Orlowski *et al.*, 2007; Stakhiev, 2000). So one of the important issues for circular saw manufacturers is to correctly determine critical rotational speed. Critical rotational speed is determined from circular saw natural frequencies which can be determined from experiments (Mote, 1965; Pahlitzs and Rowinski, 1966; Stakhiev, 2000; Stakhiev, 1998; Orlowski *et al.*, 2007; Kaczmarek *et al.*, 2015) from finite element method (FEM) analysis (Gogu, 1988; Holoyen, 1987; Leopold and Munz, 1992; Michna and Svoren, 2007) and from analytical solution for annular plates based on classical theory of thin plates (Kirchoff, 1882) where Southwell (Southwell, 1922) uses Bessel functions and Lee (Lee, 1994) polynomial functions to define mode shapes and natural frequencies. Orlowski and Sandak (Orlowski and Sandak, 2005) emphasize that permissible (max.) rotational speed of circular saw defined by the manufacturers can occasionally be higher than calculated/or experimentally determined critical rotational speed. Also, Stakhiev (Stakhiev, 2004) explicitly adduce an example where calculated permissible rotational speed is exceeded for 28% meaning that such circular saws may become unstable and consequences may not only include low quality of surface finish, but also workers injuries etc.

The purpose of this article is to analyze accuracy of classical theory of thin plates used on annular plates with analogous dimensions to circular saws while changing the clamping ratio. As an example from practice an standard clamped saw is chosen for which an FEM model is made from the producers data and for which an critical rotational speed is determined. Then critical rotational speed is calculated for annular plate with analogous dimensions to circular saw by the use of FEM model and by the use of theory for classical thin plates (mode shapes are determined with Bessel functions and by polynomial functions) and obtained results are analyzed.

## 2. Calculation of circular saw critical rotational speed

Stakhiev defines three types of rotational speeds when circular saw is stable (Stakhiev, 2000):

- universal rotational speed $n_u$=(0.31-0.43) $n_{cr}^{\min}$ ,
- optimal rotational speed $n_o$=(0.59-0.696) $n_{cr}^{\min}$ , and
- permissible rotational speed $n_p$=0.85 $n_{cr}^{\min}$ , where $n_{cr}^{\min}$ is the minimal critical rotational speed.

For circular saws there is an theory which states that resonance vibrations of circulars saws appears as an result of interference between two wave components, wave which is travelling forward and wave which is traveling backward. Based on the stated theory an equations for frequencies of backward and forward wave (Schajer, 1986) are

$$f_f = f_{s(N)} + \frac{nN}{60}, \tag{1}$$

$$f_f = f_{s(N)} - \frac{nN}{60}, \tag{2}$$

where $N$ is saws circular speed, rpm, $n$ is number of nodal diameters and $f_s$ is the natural frequency of rotating saw, Hz, which can be calculated with expression

$$f_{s(0)}^2 = f_{(N=0)}^2 + \lambda\left(\frac{N}{60}\right)^2 \tag{3}$$

where f(N=0) is the natural frequency of non-rotating saw (N=0), Hz, and $\lambda$ is an centrifugal force coefficient.

Centrifugal force coefficient λ can be determined from empirical equation (Šteuček, 1971):

$$\lambda = \frac{m_p - 1}{4m_p} n^2 + \frac{3m_p + 1}{4m_p} n \quad (4)$$

where mp is the coefficient of Poisson process which can be calculated from Poisson ratio (mp=1/$\nu$).

Circular saw will start to vibrate in a resonance when the value of backward wave frequency become zero, Figure 1 (Schajer, 1986).

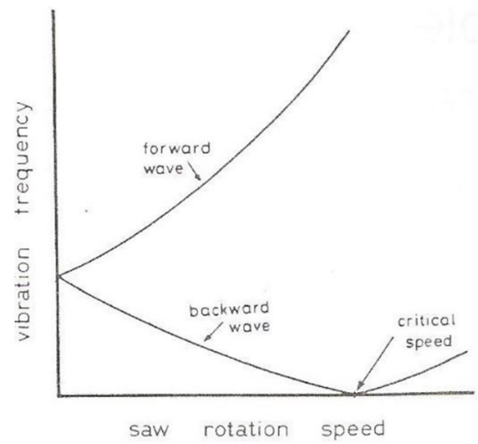


**Figure 1.** Campbell's diagram: vibration frequency of circular saw blade in function of rotational speed, an example of theory about forward and backward travelling wave (Schajer, 1986)

For natural frequency that correspond to the chosen number of nodal diameters *n* (Figure 2) the value of critical circular speed $n_{cr}$, rpm, can be determined from this equation (Kaczmarek *et al*., 2015; Stakhiev, 1998).

$$n_{cr} = \frac{60 f_{(N=0)}}{\sqrt{n^2 - \lambda}} \quad (5)$$

Critical rotational speed should be calculated for all reference modes and one should choose the lowest value for final critical speed (Figure 3). One can notice on Figure 3 that the lowest values of critical rotational speeds are connected with mode shapes with nodal diameters *n*=2, 3 and 4 which correspond to recommendations in practice.

## 3. Application of the classic theory of thin plates on annular plates

Natural frequencies and mode shapes of annular plate with analogous dimensions to chosen circular saw (thickness *h*, inner clamping diameter *a*, outer diameter which equals distance from saw center to the top of the teeth) and equal material will be determined by the use of classical theory of thin plates. Potential and kinetic energy of circular saw based on the classic theory of thin plates (Meirovitch, 1967), is determined with expressions

$$PE = \frac{D}{2}\int_0^{2\pi}\int_a^b \left( \left(\nabla^2 \psi_{mn}\right)^2 - 2(1-\nu)\left( \frac{\partial^2 \psi_{mn}}{\partial r^2}\left( \frac{1}{r}\frac{\partial \psi_{mn}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi_{mn}}{\partial \varphi^2} \right) - \left( \frac{\partial}{\partial r}\left( \frac{1}{r}\frac{\partial \psi_{mn}}{\partial \varphi} \right) \right)^2 \right) \right) r\mathrm{d}r\mathrm{d}\varphi \quad (6)$$

$$KE = \frac{\rho h}{2}\int_0^{2\pi}\int_a^b \left( \frac{\partial \psi_{mn}}{\partial t} \right)^2 r\mathrm{d}r\mathrm{d}\varphi \quad (7)$$

where bending stiffness of the blade, *D*, is defined by expression

$$D = \frac{Eh^3}{12\left(1-\nu^2\right)} \quad (8)$$

where *E* is Young's modul and $\rho$ is density.

Differential equation that describes the vibrations of circular/annular plates can be derived using Hamilton's principle. Essentially Hamilton's principle is variation of Lagrangian over time and the Lagrangian function is difference of kinetic and potential energy. The potential energy consists of work done by internal forces and work done by external forces. If there are no external forces acting on a system then the potential energy is equivalent to work done by internal forces. Kinetic energy (Baddour and Zu, 2001) is the most influential part in developing differential equation in case of stationary or rotating annular plate and potential energy is the same for both cases if there are no forces acting on a system.

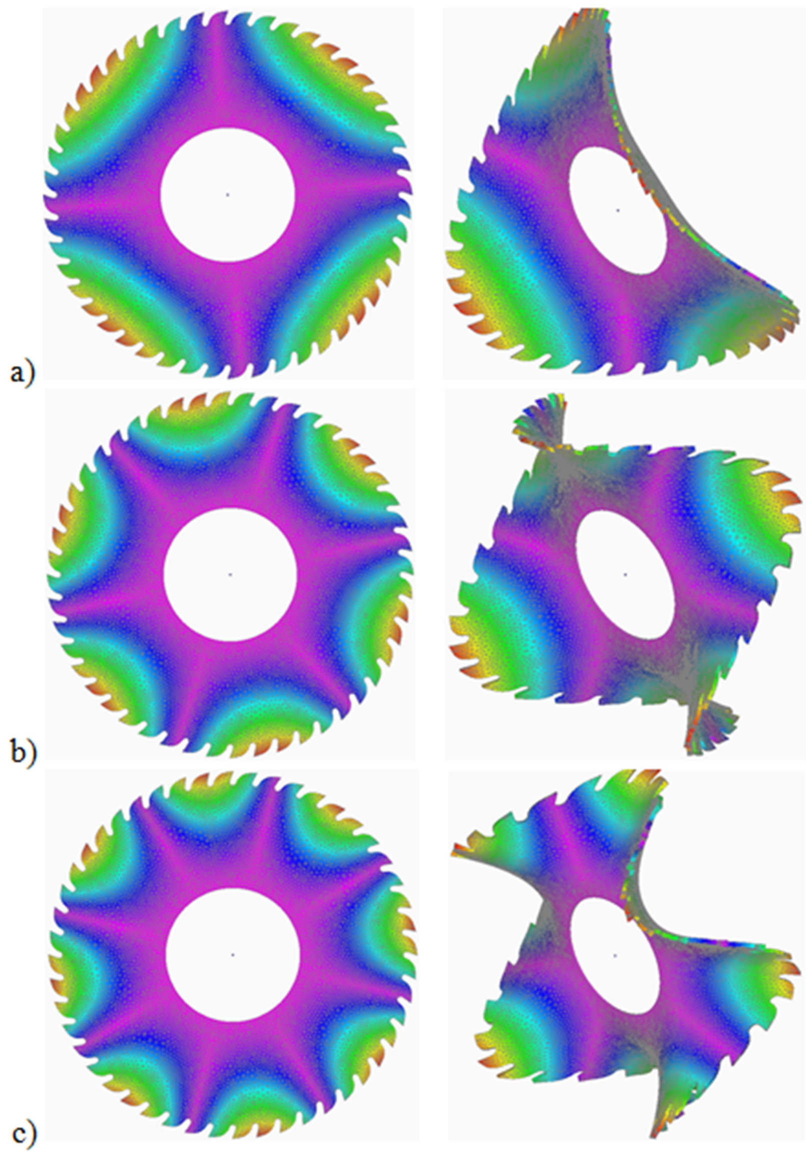


**Figure 2.** Referent mode shapes ($m$,$n$) calculated with FEM: a) (0,2), b) (0,3) and c) (0,4)

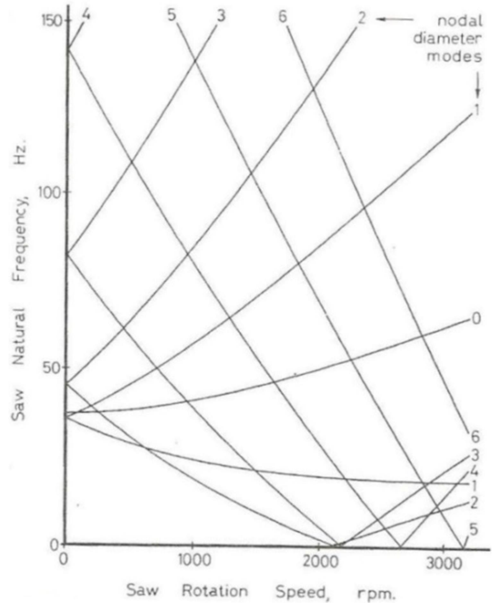


**Figure 3.** Campbell's diagram: dependence of critical rotational speed to the number of nodal diameters, (Schajer, 1986)

By applying Hamiltonian principle on to equations (6) and (7) one can calculate equation of annular plate free vibrations $w(r,\varphi,t)$

$$D\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}\right)^2 w+\rho h\frac{\partial^2 w}{\partial t^2}=0 \quad (9)$$

Boundary conditions of mounted circular saw are clamped on inner diameter ($r=a$), which is defined in this way

$$w=0\,, \quad (10)$$

$$\frac{\partial w}{\partial r}=0 \quad (11)$$

and free outer diameter which is defined in this way

$$M_r=-D\left(\frac{\partial^2 w}{\partial r^2}+\frac{v}{r}\frac{\partial w}{\partial r}+\frac{v}{r^2}\frac{\partial^2 w}{\partial \varphi^2}\right)=0, \quad (12)$$

$$Q_r+\frac{1}{r}\frac{\partial M_{r,\varphi}}{\partial \varphi}=-D\left[\frac{\partial}{\partial r}\nabla^2 w+\frac{1-v}{r^2}\frac{\partial^2}{\partial \varphi^2}\left(\frac{\partial w}{\partial r}-\frac{w}{r}\right)\right]=0 \quad (13)$$

where $M_r$ is flexural moment and $Q_r$ is transversal force. By using the method of separation of variables on equation (9) the solution takes the following form:

$$w(r,\varphi,t)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\psi_{m,n}(r)\cos(n\varphi)e^{i\omega_{m,n}t} \quad (14)$$

where mode shape $\psi_{m,n}$ will be defined with Bessel functions and with polynomial functions.

### 3.1. Mode shapes expressed with Bessel functions

Mode shape expressed with Bessel functions must satisfy boundary conditions (10-13) and they are defined with (Southwell, 1922; Meirovitch, 1967).

$$\psi_{m,n}(r,\varphi)=\cos(n\varphi)\left(A_{m,n}J_n\left(\frac{\beta_{m,n}r}{b}\right)+B_{m,n}Y_n\left(\frac{\beta_{m,n}r}{b}\right)+C_{m,n}I_n\left(\frac{\beta_{m,n}r}{b}\right)+D_{m,n}K_n\left(\frac{\beta_{m,n}r}{b}\right)\right), \tag{15}$$

where coefficients $A_{m,n}, B_{m,n}, C_{m,n}$ and $D_{m,n}$ follows from specific mode shape, $J_n$ and $Y_n$ are the Bessel functions of first kind and the second kind, respectively, while $I_n$ and $K_n$ are modified Bessel functions of first and second kind and dimensionless frequency parameter $\beta_{mn}$ is defined as

$$\beta_{mn}=\sqrt[4]{\frac{\omega_{mn}^2\rho h}{D}}\ . \tag{16}$$

All coefficients are determined from the boundary conditions by applying the energy principle (Meirovitch, 1967). Natural frequency can be determined from dimensionless frequency parameter using following expression

$$f_{m,n}=\frac{1}{2\pi}\left(\frac{\beta_{m,n}^2}{b^2}\sqrt{\frac{D}{\rho h}}\right) \tag{17}$$

### 3.2. Mode shapes expressed with polynomial function

Mode shapes expressed with polynomial functions must satisfy boundary conditions and is defined as (Lee, 1994):

$$\psi_{mn}(r,\varphi)=\cos(n\varphi)\sum_{s=2}^{P(m)}c_{mn,s}(r-a)^s \tag{18}$$

Total potential and kinetic energy of the system are obtained by entering modal functions which includes boundary conditions in to equations (6) and (7) (Bert, 1987) and by applying energy principle one obtains an sum of algebraic equations which can be written in matrix form (Meirovitch, 1967; Bert, 1987; Kim *et al*., 1990)

$$\begin{bmatrix}\Lambda_{22} & \Lambda_{23} & \dots & \Lambda_{2N}\\ \Lambda_{32} & \Lambda_{33} & \dots & \Lambda_{3N}\\ \dots & \dots & \dots & \dots\\ \Lambda_{N2} & \Lambda_{N3} & \dots & \Lambda_{NN}\end{bmatrix}\begin{bmatrix}c_{mn,2}\\ c_{mn,3}\\ \dots\\ c_{mn,4}\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix} \tag{19}$$

where $\Lambda_{ij}$ is defined as

$$\Lambda_{ij}=ij(i-1)(j-1)X_1^{i+j-4}+\nu ij(i+j-2)X_0^{i+j-3}+\left(ij\left(2(1-\nu)n^2+1\right)-\nu n^2\left(i^2+j^2-i-j\right)\right)X_{-1}^{i+j-2}$$
$$-(3-2\nu)n^2(i+j)X_{-2}^{i+j-1}+n^2\left(n^2+2(1-\nu)\right)X_{-3}^{i+j}-\beta_{mn}^4X_1^{i+j} \tag{20}$$

and $X_k^l$ is defined as

$$X_k^l = \int_a^b (r-a)^l r^k \mathrm{d}r \tag{21}$$

or

$$X_k^l = \begin{cases} \dfrac{(b-a)^{l+1}}{l+1}, \ k=0 \\ (b-a)^{l+1}\left(\dfrac{b-a}{l+2}+\dfrac{a}{l+1}\right), \ k=1 \\ \dfrac{l!(-a)^{l+k+1}\ln\left(\dfrac{b}{a}\right)}{(l+k+1)!(-k-1)!}+\displaystyle\sum_{s=0,s\neq l+k+1}^{l}\dfrac{l!(-a)^s\left(b^{l+k-s+1}-a^{l+k-s+1}\right)}{(l+k-s+1)(l-s)!s!}, \ k<0 \end{cases} \tag{22}$$

Dimensionless frequency parameter $\beta_{mn}$ is than calculated from the fact that quadratic matrix determinant must be zero to find an solution. Then, natural frequency can be determined from dimensionless frequency parameter using expression (17).

## 4. Examples

Saw teeth dimensions are taken from the literature (Nishio and Marni, 1996) and are shown on the Figure 4. Dimensions of the circular saw blade disk are taken for the circular saw Bosch 2609256883 (Precision Circular Saw Blade with 48 Carbide Teeth, 300 mm Diameter, 30 mm Bore, 3.2 mm Cutting Width) without slots and are shown on Figure 5.

### 4.1. Example 1

The purpose of the Example 1 is to define and compare natural frequencies of standard circular saw without slots and annular plate with analogous dimensions by the use of FEM. Input data for annular plate (Figure 6) are listed in the Table 1.

**Table 1.** Geometric characteristics of annular plate

| Geometrical characteristics | |
|---|---|
| Outer radius b (mm) | 150 |
| Inner clamping radius a (mm) | 55 |
| Thickness h (mm) | 2,2 |

It can be seen from the Table 2 that calculated natural frequencies converge to the value with minimum element size 0.003 m. Also it can be seen that the percentage difference between results of annular plates and circular saws for referent mode shapes (*m*, *n*) are: (0,2) 0.62%, (0,3) 3.57% i (0,4) 7.25%. The results are in good agreement for the first two referent modes but for the third mode there is a bit bigger deviation. One can conclude that results would be better if circular saws with smaller teeth were used.

In Table 3 there are natural frequency of circular saw blade and annular plate with analogous dimensions calculated by FEM.

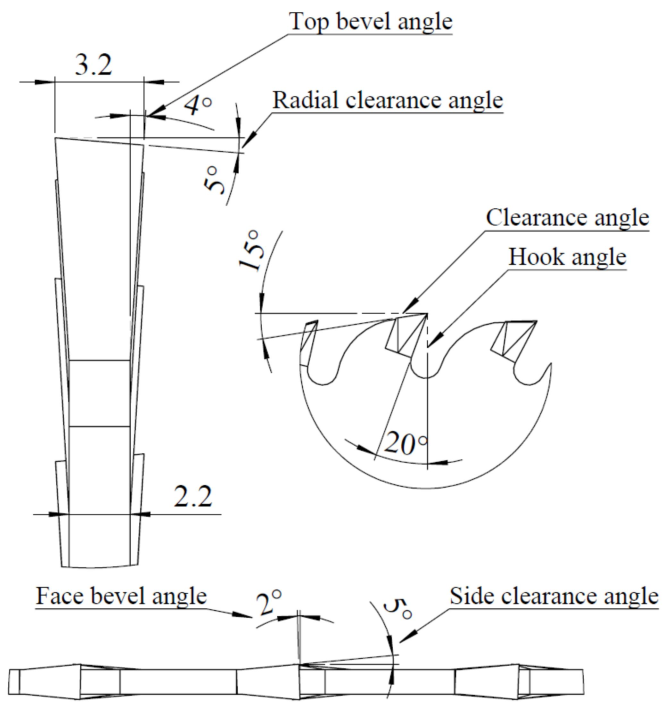


**Figure 4.** Teeth geometry of circular saw blade (dimensions are in mm)

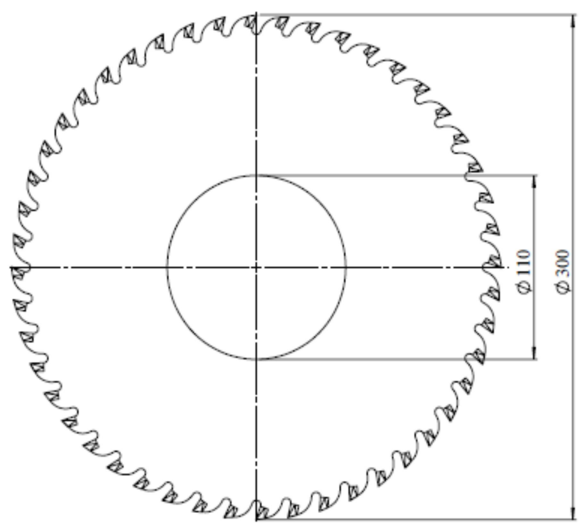


**Figure 5.** Geometry of circular saw blade (dimensions are in mm), $\phi$ 110 mm is the clamping collar outer diameter

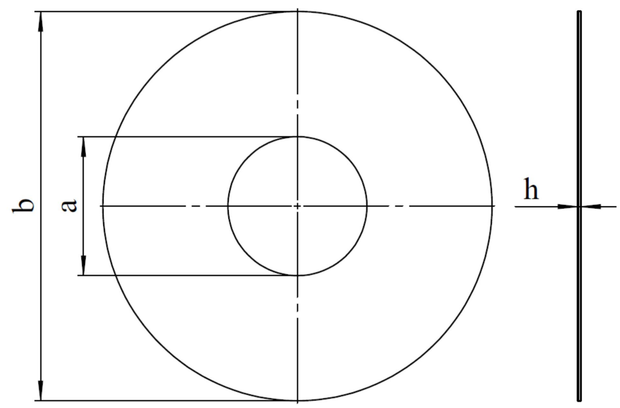


**Figure 6.** Dimensions of annular plate

**Table 2.** Natural frequency of circular saw blade and annular plate obtained by using FEM in Femap NX Nastran

| Mode Shape (m,n) | Natural frequency of circular saw blade [Hz] (Element size 0.008) | Natural frequency of circular saw blade [Hz] (Element size 0.005) | Natural frequency of circular saw blade [Hz] (Element size 0.003) | Natural frequency of annular plate [Hz] (Element size 0.008) | Natural frequency of annular plate [Hz] (Element size 0.005) | Natural frequency of annular plate [Hz] (Element size 0.003) |
|---|---|---|---|---|---|---|
| (0,0) | 198.90 | 198.30 | 197.82 | 195.67 | 194.78 | 194.41 |
| (0,1) | 200.55 | 199.95 | 199.50 | 196.94 | 195.81 | 195.39 |
| (0,2) | 231.35 | 230.22 | 229.78 | 230.31 | 228.84 | 228.36 |
| (0,3) | 334.77 | 331.81 | 331.08 | 345.02 | 343.37 | 342.90 |
| (0,4) | 513.49 | 507.03 | 505.55 | 545.21 | 542.83 | 542.29 |

**Table 3.** Natural frequency of annular plate obtained by different methods

| Mode Shape (m,n) | Frequency obtained using Femap NX Nastran for annular plate [Hz] (Element size 0.003) | Frequency obtained using linear vibration theory of annular plate, Bessel [Hz] | Frequency obtained using linear vibration theory of annular plate, polynomial s=6 [Hz] |
|---|---|---|---|
| (0,0) | 194.41 | 194 | 194 |
| (0,1) | 195.39 | 189.9 | 195 |
| (0,2) | 228.36 | 228.24 | 228.3 |
| (0,3) | 342.9 | 343.29 | 343.4 |
| (0,4) | 542.3 | 543.43 | 543.5 |

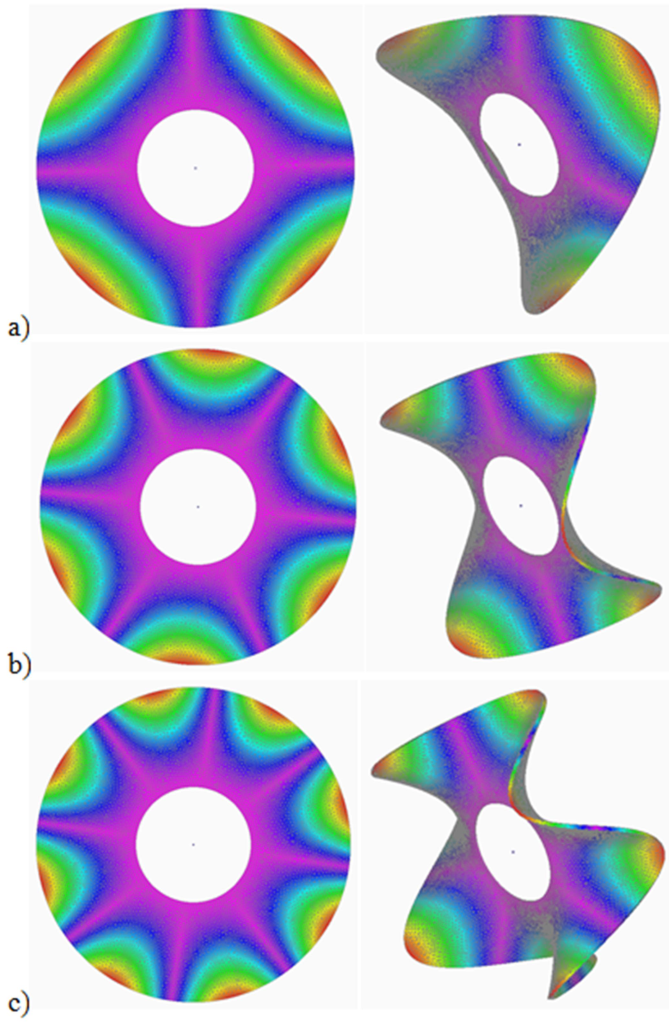


**Figure 7.** Referent annular plate mode shapes ($m$, $n$) clamped at the inner radius: a) (0,2), b) (0,3) i c) (0,4)

On the figure there are referent annular plate mode shapes and the similarity with circular saws mode shapes can be seen (Figure 7).

### 4.2. Example 2

The purpose of Example 2 is to calculate natural frequencies with the use of classical theory of thin plates on annular plates and to compare it with the FEM results (Table 3).
It can be seen that calculated natural frequencies from classical theory of thin plates completely match FEM natural frequencies for annular plate. The percentage difference for mode shapes ($m$, $n$) are: (0,2) 0.03%, (0,3) 0.15% and (0,4) 0.22%.

### 4.3. Example 3

The purpose of the Example 3 is to analyze convergence of annular plates natural frequencies when mode shapes are defined with polynomial functions (18) (Table 4).

**Table 4.** Convergence of natural frequency with increasing of polynomial order, Hz

| s | (0, n) | | | | | |
|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 |
| 2 | 287 | 287.1 | 307.2 | 391.3 | 563.1 | 817.3 |
| 3 | 201 | 203.2 | 237.6 | 350.4 | 547.1 | 814.7 |
| 4 | 194.2 | 195.4 | 229.2 | 344.4 | 544.3 | 814 |
| 5 | 194 | 195.1 | 228.5 | 343.8 | 544.1 | 814 |
| 6 | 194 | 195 | 228.3 | 343.4 | 543.5 | 813.3 |

Maximum number of polynomial functions that can be used to define mode shapes, *P*, is 8 (Lee, 1994). Number of polynomial functions needed to define mode shape is in direct correlation with the number of modal circles in mode shapes. In used referent mode shapes number of modal circles is zero, while the number of nodal diameters is 2 to 4 which associate to possibility that relatively small polynomial order *s* can give good accuracy (ie. percentage differences between referent natural frequencies with polynomial order *s*=3 and *s*=6 (see Table 5) for referent modal shapes (*m*,*n*) are: (0,2) 4%, (0,3) 2.04% i (0,4) 0.66% ).

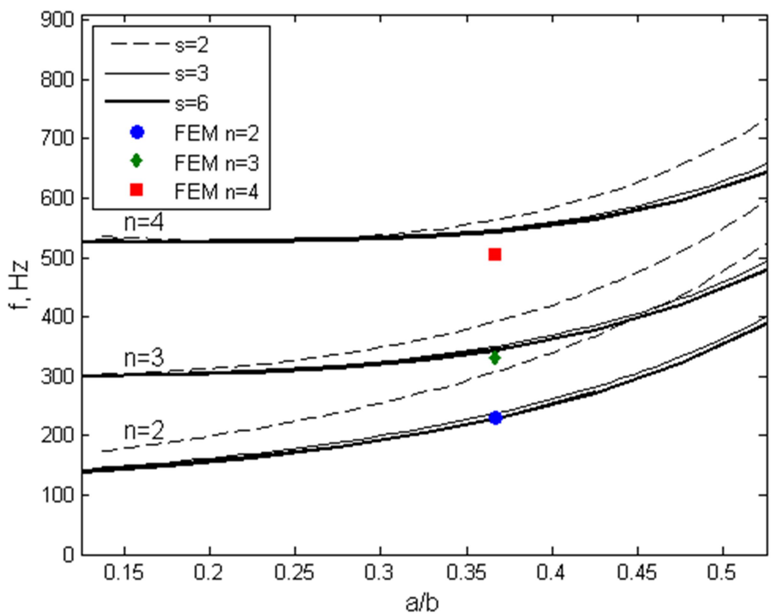


**Figure 8.** Natural frequencies of mode shapes *m*=0, *n*=2,3 i 4, for annular plate calculated with polynomial functions (curves) and for circular saw calculated with FEM (markers)

On the Figue 8 there are values of different natural frequencies as a function of clamping ratio (*a*/*b*) where one can see that *s*=3 has pretty high accuracy through the whole clamping ratio domain.

### 4.4. Example 4

The purpose of Example 4 is to analyze values of critical rotational speeds for referent mode shapes *m*=0, *n*=2,3 and 4, as a function of clamping ratio (*a*/*b*), Figure 9.

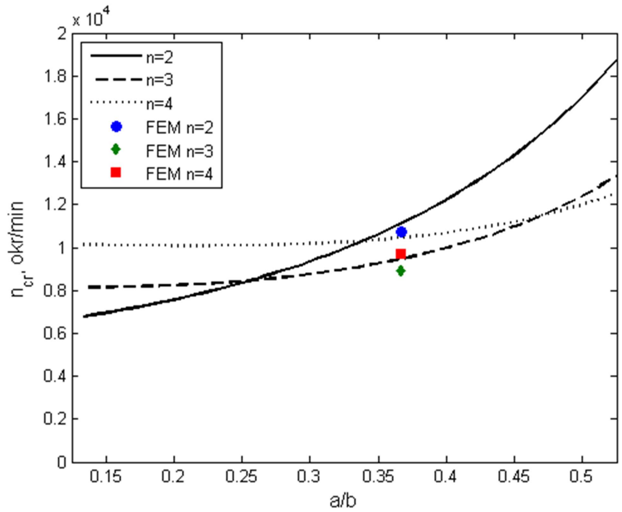


**Figure 9.** Critical rotational speed for referent mode shapes *m*=0, *n*=2,3 i 4, for annular plate calculated with polynomial functions (curves) and for circular saw calculated with FEM (markers)

On Figure 9 there are values of critical rotational speed for referent mode shapes *m*=0, *n*=2,3 i 4, for annular plate calculated with polynomial functions (*s*=3) and for circular saw calculated with FEM. It can be seen that clamped ratio domain is devided in to bands where minimum critical speed follows from mode shape (0,2), than from mode shape (0,3) and at the end from mode shape (0,4). That kind of behavior of critical rotational speed is expected and that is why it is recommended in practice to calculate critical rotational speed for all referent mode shapes and than to choose the minimum one. On the Figure 9 there are also values of critical rotational speeds calculated with FEM model for circular saw. It can be seen that percentage difference exists and the value is 5.83%. The value would be smaller if smaller teeth were used.

## 5. Conclusions

In this article an procedure for calculation of critical rotational speed of standard clamped circular saw (with flat disk surfaces and without slots) is described. Critical rotational speeds are calculated from the values of natural frequencies which are calculated with FEM and classic theory of thin plates used on annular plate with analogous dimensions. In the classic theory of thin plates mode shapes are defined with Bessel functions and with polynomial functions. The convergence of the natural frequencies calculated with polynomial functions are analyzed and an optimum accuracy is chosen for calculation of critical rotational speed. It is seen that there is high percentage differences between natural frequencies for mode shape (0,4) calculated with FEM for circular saw and polynomial expressions for annular plate which is connected with relatively big teeth. I one chooses smaller teeth it is expected that natural frequencies calculated for annular plate would have better accuracy. Also critical rotational speeds are calculated from the annular plates with modes defined by polynomial functions with defined lower polynomial order and results are shown in clamped ratio domain. It can be concluded that for the chosen circular saw one can predict critical rotational speeds for clamping ratio domain which ends with minimium for nodal diameter *n*=3. Higher accuracy for higher clamping ratio domain is expected for standard circular saws with smaller teeth.

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