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MEASUREMENT ERROR EFFECT ON THE POWER OF THE CONTROL CHART FOR ZERO-TRUNCATED BINOMIAL DISTRIBUTION UNDER STANDARDIZATION PROCEDURE

Abstract: *Measurement error effect on the power of control charts for zero truncated Poisson distribution and ratio of two Poisson distributions are recently studied by Chakraborty and Khurshid (2013a) and Chakraborty and Khurshid (2013b) respectively. In this paper, in addition to the expression for the power of control chart for ZTBD based on standardized normal variate is obtained, numerical calculations are presented to see the effect of errors on the power curve. To study the sensitivity of the monitoring procedure, average run length (ARL) is also considered.*

Keywords: *power, zero-truncated binomial distribution (ZTBD), measurement error, average run length (ARL)*

1. Introduction

Binomial distribution is used to construct control chart for attributes, either p-chart or d-chart when fraction defective or the number of defective is concerned. Probability distributions often arise in practice which are of binomial type, but for some reason zero value is unobserved. For example, suppose that the variable understudy represents the number of defective items in a manufactured lot of n items and r defects are inevitable and not more than n are observed, then $x = r, r+1, \cdots, n$ and may follow a singly truncated binomial distribution. A special case, when $r = 1$ means zero-truncated or positive binomial distribution (ZTBD) which is dealt with in this paper. The significance of ZTBD is illustrated by Johnson *et al*. (2005) with real-life applications of truncated binomial distribution. Chakraborty and Khurshid (2011) constructed one-sided cumulative sum control charts for ZTBD and extended their study for doubly truncated binomial distribution when the underlying distribution is the ratio of two Poisson distributions (Chakraborty and Khurshid, 2012).

An interesting example of a practical application of ZTBD has been described by Biswas and Sriwastav (2011): An electronic device has n transistors and the device will flash red light if one or more transistors fail. When the red light flashes the device is examined to detect the faulty transistors. During a specified period of operation, the problem is to obtain the probability of exactly r faulty transistors, if each of the transistors has independently a probability p of failing. Now for the detection of faulty transistors the red light must flash i.e., there must be one or more faulty transistors. If X (a random variable) represents the number of

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faulty transistors, then $X \sim bin(n, p)$. The required probability is $P[X = x | X > 0]$, which is given by the zero-truncated binomial distribution.

Many studies would assume that the measurement is without error and is a significant issue. Measurement errors, which often exist in practice, may considerably affect the performance of control charts (Ryan, 2011). Often the process variability is observed in any control chart which is the combination of inherent variability in the processes and the error due to the measurement instrument. Kanazuka (1986) observed that if the measurement error is large relative to the process variability, the control chart to detect any shift in the process level is affected. The sources of error may be due to inherent variability in the process and the error due to measurement instrument. The efficiency and the ability of the control chart to detect the shift of the process level will be affected if the measurement error is large relative to the process variability.

There has been considerable research, in recent years where the actual performance of various control charts in the presence of measurement error is examined. We now briefly review the status of the research on the subject to measurement error. The effect of measurement errors in $\overline{X}$ chart was recognized early on in control chart construction by Bennett (1954). This seminal work was followed by Mizuno (1961), Abraham (1977) and Mittag and Stemann (1998). Singh (1964) studied measurement error in acceptance sampling for attributes. Kanazuka (1986) and Mittag (1995) considered the effect of measurement error on the power of the $\overline{X} - R$ control charts. Rahim (1985) investigated the effect of non-normality and measurement errors on the economic design of charts. Walden (1990) measured the power of $\overline{X}$, R and $\overline{X} - R$ charts using *ARL* when measurement error affects the system. Smith (1990) considered measurement error and its effect on the probability of making correct decisions regarding acceptance of product, and consequently, on the costs associated with the inspection process. Linna (1991) exhibited the effect of increasing the measurement variance and slope of covariate model on Shewhart control charts. Tricker *et al.* (1998) investigated the effects of one particular aspect of measurement error (round-off) on *R* control chart. Moreover, (Linna and Woodall, 2001; Linna *et al*., 2001) showed the effect of measurement error on Shewhart control charts using a linear covariate and multivariate control charts respectively.

Stemann and Weihs (2001) and Maravelakis *et al.* (2004) investigated the effect of measurement error on the EWMA chart. Shore (2004) pointed out the requirements of measurement error, to satisfy the various control charts. Yang (2002) presented the effect of measurement error on the asymmetric economic design and *S* control charts. Chang and Gan (2006) proposed Shewhart chart for monitoring the linearity between two measurement gauges. Huwang and Hung (2007) described the effect of measurement error on the control charts for monitoring multivariate process variability. Yang *et al*. (2007) considered a process model to take into account of measurement error on two dependent processes (Yang and Yang, 2005). Xiaohong and Zhaojun (2009) demonstrated the effect of measurement error on the CUSUM chart for the autoregressive data. Costa and Castagliola (2011) exhibited the effect of measurement error and autocorrelation on the $\overline{X}$ chart. Moameni *et al.* (2012) examined the effect of measurement error on the effectiveness of the fuzzy control chart to detect out of control situations. Maravelakis (2012) retriated the old problem and investigated the effect of measurement error on the performance of the CUSUM control chart for the mean. Sankle *et al.* (2012) studied CUSUM control charts for truncated normal distribution under measurement error. Recently, Yang *et al.*, (2013) derived a new

EWMA control chart to monitor the exponentially distributed service time between consecutive events with the measurement error instead of monitoring the number of events in a given time interval. The performance of the synthetic chart was investigated by Hu *et al.*, (2014) when measurement errors exist using a linearly covariate error model.

Recently, measurement error effect on the power of control charts for zero truncated Poisson distribution and ratio of two Poisson distributions were studied by Chakraborty and Khurshid (2012) and Chakraborty and Khurshid (2013) respectively. In this paper, in addition to the expression for the power of control chart for ZTBD based on standardized normal variate is obtained, numerical calculations are presented to see the effect of errors on the power curve. To study the sensitivity of the monitoring procedure, average run length (*ARL*) is also considered.

2. Zero-truncated Binomial Distribution (ZTBD)

A Zero-truncated Binomial Distribution (ZTBD) is a modified form of a binomial distribution. A random variable *X* is said to follow ZTBD if it assumes only non-negative values and its probability mass function is given by

$$f_{X|\{(n,p)\}}(d) = \binom{n}{d} p^d q^{n-d} (1-q^n)^{-1} \quad (1)$$

where $d \in \{1, 2, ..., n\}$.

The first moment and variance of $X\left|\{(n,p)\}\right.$ are

$$E\left[X\left|\{(n,p)\}\right.\right] = \mu = np(1-q^n)^{-1}, \quad (2)$$

and

$$Var\left[X\left|\{(n,p)\}\right.\right] = \sigma^2 = \frac{1}{1-q^n}\left[npq + n^2p^2 - \frac{n^2p^2}{1-q^n}\right] \quad (3)$$

3. Assumptions and notations

In the development of the power of the control chart and *ARL* for equation (1), the following assumptions are made and notations are used:

i. The measurement of items is used to ascertain the number of defects in a lot.
ii. The process has binomial distribution with mean μ and variance σ_P^2.
iii. The applied measurement process (which is independent of the manufacturing process) has a variance σ_m^2. Thus, the overall variability is given by $\sigma^2 = \sigma_p^2 + \sigma_m^2$.
iv. Measurements of the items are taken to classify the produced units into defective and non-defective ones.
v. The process is in a state of statistical control at the time of determining the control limits and the same measuring instrument is used for later measurements;
vi. When the process parameter shifts, the data is restricted from a binomial distribution, however, with mean μ' and variance $(\sigma_{p'}^2 + \sigma_m^2)$ where $\sigma_{P'}^2$ is the process variance when the parameter shifts (For details see Chakraborty and Khurshid, 2013 a, b).

Thus, considering the above assumptions, Shewhart 3σ control limits will be $\mu \pm K\sqrt{\sigma_p^2 + \sigma_m^2}$. Normally we chose *K*=3as it will give no false alarm with probability of atleast 99.73% (Montgomery, 2013).

Let $D\left|\{(n,p)\}\right.$ be the number of defective items. In a sample of size n, which is binomial variable with parameters n and p. If the sample proportion of defective item is plotted with Shewhart 3σ control limits with mean μ and variance $\sigma^2 = \sigma_p^2 + \sigma_m^2$, then following Chakraborty and Khurshid (2013 b), the power of detecting the change of process parameter for the control chart for fraction defective under measurement error is given by

$$P_p = P_{D|\{(n,p)\}}\left(\left\{d \middle| d \geq \mu + K\sqrt{\sigma_p^2 + \sigma_m^2}\right\}\right) + \left(\left\{d \middle| d \leq \mu + K\sqrt{\sigma_p^2 + \sigma_m^2}\right\}\right) \tag{4}$$

4. Power of control chart for standardized zero truncated binomial variables

Instead of plotting the number of defects (or fraction defectives) in the control chart, we can standardize the variables as given below and plot accordingly. This standardization procedure not only stabilizes the variables, but also stabilizes the resulting control chart. In this case the control limits as well as central lines are invariant with sample size n.

Thus, equation (4) can be expressed in terms of standardized normal variable Z (when sample size is large and varies):

$$Z\left|\{(\mu_p, \sigma_p^2, \sigma_m^2, n)\}\right. = \frac{D\left|\{(\mu_p, \sigma_p^2, \sigma_m^2, n)\} - \mu_p\right.}{\sqrt{(\sigma_p^2 + \sigma_m^2)}} \tag{5}$$

where

$$\mu_p = \frac{n\,p_p}{\{1-(1-p_p)^n\}},$$

$$\sigma_p^2 = \frac{1}{\{1-(1-p_p)^n\}}\left[n\,p_p(1-p_p) + n^2 p_p^2 + \frac{n^2 p_p^2}{\{1-(1-p_p)^n\}}\right]$$

and

$$\sigma_m^2 = \frac{1}{\{1-(1-p_m)^n\}}\left[n\,p_m(1-p_m) + n^2 p_m^2 + \frac{n^2 p_m^2}{\{1-(1-p_m)^n\}}\right].$$

Hence, following Kanazuka (1986), Chakraborty and Khurshid (2013 b) and using equation (5), when the process parameter changes from μ to μ', the power of the control chart for equation (1) is

$$P_d = P_{D|\{(\mu,\mu',\sigma_p^2,\sigma_{p'}^2,\sigma_m^2,n)\}}\left\{\left(d\left|\frac{d-\mu'}{\sqrt{(\sigma_{p'}^2+\sigma_m^2)}} \geq \frac{(\mu-\mu')}{\sqrt{(\sigma_{p'}^2+\sigma_m^2)}} + 3\frac{\sqrt{\sigma_p^2+\sigma_m^2}}{\sqrt{(\sigma_{p'}^2+\sigma_m^2)}}\right.\right)\right\}$$
$$+ P_{D|\{(\mu,\mu',\sigma_p^2,\sigma_{p'}^2,\sigma_m^2,n)\}}\left\{\left(d\left|\frac{d-\mu'}{\sqrt{(\sigma_{p'}^2+\sigma_m^2)}} \leq \frac{(\mu-\mu')}{\sqrt{(\sigma_{p'}^2+\sigma_m^2)}} - 3\frac{\sqrt{\sigma_p^2+\sigma_m^2}}{\sqrt{(\sigma_{p'}^2+\sigma_m^2)}}\right.\right)\right\}$$

$$= P_{Z\left|\{(\mu,\mu',\sigma_p^2,\sigma_{p'}^2,\sigma_m^2,n)\}\right.}\left\{\left(Z\left|Z \geq \sqrt{\frac{1+R^2}{(K^2+R^2)}}\left[3-\frac{d}{\sqrt{(1+K^2)}}\right]\right.\right)\right\}$$

$$+ P_{Z\left|\{(\mu,\mu',\sigma_p^2,\sigma_{p'}^2,\sigma_m^2,n)\}\right.}\left\{\left(Z\left|Z \leq \sqrt{\frac{1+R^2}{(K^2+R^2)}}\left[-3-\frac{d}{\sqrt{(1+K^2)}}\right]\right.\right)\right\}$$

$$= \Phi\left\{\sqrt{\frac{1+R^2}{(K^2+R^2)}}\left(-3+\frac{d}{\sqrt{1+R^2}}\right)\right\}+\Phi\left\{\sqrt{\frac{1+R^2}{(K^2+R^2)}}\left(-3-\frac{d}{\sqrt{1+R^2}}\right)\right\} \quad (6)$$

$$= \Phi(A)+\Phi(B)$$

where $d = \{(\mu'-\mu)/\sigma_p)\}$, $K^2 = \sigma_{p'}^2/\sigma_p^2$, $R^2 = \sigma_m^2/\sigma_p^2$,

$$A = \left\{\sqrt{\frac{1+R^2}{(K^2+R^2)}}\left(-3+\frac{d}{\sqrt{1+R^2}}\right)\right\},\ B = \left\{\sqrt{\frac{1+R^2}{(K^2+R^2)}}\left(-3-\frac{d}{\sqrt{1+R^2}}\right)\right\}$$

and

$$\Phi(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-(u^2/2)}du.$$

The power of the control chart P_d can be obtained easily by solving $\Phi(z)$ for different combinations of d K^2 and R^2. These are shown in Tables 1-4.

5. Average Run Length (*ARL*) for ZTBD under measurement error

To study the sensitivity of the monitoring procedure, one can also study *ARL* which is the average number of points that must be plotted before a point indicates an out of control condition when operating is statistical control.

For any Shewhart control chart, the $ARL = [P]^{-1}$ where P is the probability that a single point exceeds the control limits. In this, one can interpret the results of the power of control chart in terms of *ARL* just by reversing equation (6) rather than drawing conclusions based on P_d .

$$ARL = \left[\Phi\left\{\sqrt{\frac{1+R^2}{(K^2+R^2)}}\left(-3+\frac{d}{\sqrt{1+R^2}}\right)\right\}+\Phi\left\{\sqrt{\frac{1+R^2}{(K^2+R^2)}}\left(-3-\frac{d}{\sqrt{1+R^2}}\right)\right\}\right]^{-1} \quad (7)$$

The values of *ARL* are shown in Table 5.

6. Concluding remarks

The measurement effect on the power of detecting the changes in the process parameter by Shewhart 3σ control limits with the control chart for ZTBD is shown in Tables 1 to 5.

It has been observed from Table 1 that increase in the shift of the process parameter μ to μ', there is also an increase in the power of control chart P_d for fixed values of *n*, *P* and σ_m^2.

Table 1. Power of control chart (when *n=10*, *p*=0.3, $\sigma_m^2 = 0.02$)

$\mu = 3.09$, $\sigma_p^2 = 1.88$, $R^2 = \sigma_m^2 / \sigma_p^2 = 0.01$							
p'	μ'	$\sigma_{p'}^2$	$d = \frac{\mu' - \mu}{\sigma_p}$	$K^2 = \frac{\sigma_{p'}^2}{\sigma_p^2}$	$\Phi(A)$	$\Phi(B)$	P_d
0.4	4.02	2.30	0.68	1.23	0.02	0.0005	0.021
0.5	5.0	2.48	1.39	1.32	0.0808	0.0001	0.0809
0.6	6.0	2.18	2.12	1.15	0.20	0.025	0.225
0.7	7.0	2.10	2.85	1.1	0.44	0.0001	0.4401
0.8	8.0	1.60	3.58	0.85	0.72	0.0001	0.7201

Thus smaller the change in the process average, the smaller the power of test. The values of K^2 also affect the power P_d of the control chart. Smaller values of K^2 corresponds to the larger values of P_d.

From Tables 1 and 2, it has been observed that the values of P_d considerably increase as we go on increasing the value of *n* for fixed *p* and σ_m^2.

Table 2: Power of control chart (when *n=15*, *p=0.3*, $\sigma_m^2 = 0.02$)

$\mu = 4.52$, $\sigma_p^2 = 3.065$, $R^2 = \sigma_m^2 / \sigma_p^2 = 0.0065$							
p'	μ'	$\sigma_{p'}^2$	$d = \frac{\mu' - \mu}{\sigma_p}$	$K^2 = \frac{\sigma_{p'}^2}{\sigma_p^2}$	$\Phi(A)$	$\Phi(B)$	P_d
0.4	6.0	3.58	1.44	1.168	0.0749	0.0001	0.075
0.5	7.5	3.75	1.70	1.22	0.1210	0.0001	0.1211
0.6	9.0	3.60	2.56	1.175	0.3409	0.0001	0.341
0.7	10.5	3.15	3.42	1.03	0.6544	0.0001	0.6555
0.8	12.0	2.40	4.27	0.78	0.9222	0.0001	0.9233

Change in the value of n also affect the relative measurement error R^2. Greater the value of n, smaller will be the value of R^2 and smaller value of R^2 results higher magnitude of P_d.

Table 3 shows the values of P_d for fixed values of n, P and σ^2_P. It is seen from the table that for fixed σ^2_m and R^2, the values of P_d increase as there is an increase in deviation from μ to μ'. On the contrary we can say that smaller the values of K^2 larger will be the values of P_d. But for fixed deviation, the values of P_d decrease as we go on increasing the values of relative measurement error R^2

Table 4 gives the values of the power of the control chart P_d corresponding to the values of σ^2_m. Larger the values of measurement error, smaller the detecting power, however this effect is small if n and d are large enough.

Table 3. Values of P_d **(**when $\sigma_p^2 = 1.88, n = 10, p = 0.3, \mu = 3.09$)

		σ^2_m						
		0.02	0.05	0.1	0.2	0.3	0.5	
		R^2						
μ'	K^2	0.01	0.03	0.05	0.11	0.16	0.27	d
4.02	1.23	0.0188	0.0184	0.0175	0.0167	0.0159	0.0144	0.68
5.0	1.32	0.0794	0.0779	0.075	0.0695	0.0656	0.0572	1.39
6.0	1.15	0.2039	0.1978	0.1923	0.1763	0.1697	0.1447	2.12
7.0	1.12	0.4365	0.4287	0.4169	0.3898	0.3670	0.3121	2.85
8.0	0.85	0.7292	0.7158	0.7020	0.6665	0.6369	0.5754	3.58

Table 4. Power of control chart

σ^2_m	R^2	$\Phi(A)$	$\Phi(B)$	P_d
0.02	0.01	0.0808	0.0001	0.0809
0.05	0.03	0.0793	0.0001	0.0794
0.10	0.05	0.0749	0.0001	0.0750
0.20	0.11	0.0694	0.0001	0.0695
0.30	0.16	0.0655	0.0001	0.0656
0.50	0.27	0.0571	0.0001	0.0572

0.70	0.37	0.0516	0.0001	0.0517

where, $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-(u^2/2)} du$ is the standard normal distribution.

It has also been observed from the above tables that relative measurement errors R^2 tend to increase along with the power of the control chart when there is a change (increase) in the process average.

Table 5: Values of *ARL* **(**when $\sigma_p^2 = 1.88, n = 10, p = 0.3, \mu = 3.09$)

		σ_m^2						
		0.02	0.05	0.1	0.2	0.3	0.5	
		R^2						
μ'	K^2	0.01	0.03	0.05	0.11	0.16	0.27	d
4.02	1.23	53.19	54.33	57.14	59.88	62.89	69.44	0.68
5.0	1.32	12.59	12.84	13.33	14.39	15.24	17.48	1.39
6.0	1.15	4.90	5.06	5.20	5.67	5.89	6.91	2.12
7.0	1.12	2.29	2.33	2.40	2.56	2.72	3.20	2.85
8.0	0.85	1.37	1.40	1.42	1.50	1.57	1.74	3.58

Table 5 gives the values of *ARL*. It has been observed from the table that values of *ARL* tend to decrease as the change in the process parameter (d) increase for fixed values of n, p, σ_P^2 and R^2, where as for fixed deviation and fixed K^2, *ARL* values tend to increase as the values of σ_m^2 increase which indicates that presence of measurement error delay the detection process of the change in the process level.

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