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THE PROCESS CAPABILITY ANALYSIS - A TOOL FOR PROCESS PERFORMANCE MEASURES AND METRICS - A CASE STUDY

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Abstract: *Process Capability can be evaluated through the computations of various process capability ratios and indices. The basic three capability indices commonly used in manufacturing industries are Cp, Cpk,Cpm and Cpmk .Process capability indices are intended to provide single number assessment of the ability of a process to meet specification limits on quality characteristics of interest. Thus, it identifies the opportunities for improving quality and productivity. The level of significance on process capability analysis has been increased considerably over last decade, but the literature findings reveal the importance of understanding the concepts, methodologies and critical assumptions while its implementation in manufacturing process. The objective of this paper is to conduct process capability analysis for boring operation by understanding the concepts, methodologies and making critical assumptions.*

Keywords: *Process Capability Index, Normal Distribution, Statistical Process Control, Run chart*

1. Introduction

Process capability study is a method of combining the statistical tools developed from the normal curve and control charts with good engineering judgment to interpret and analyze the data representing a process. The purpose of the process capability study is to determine the variation spread and to find the effect of time on both the average and the spread. The administration, analysis and use of the process capability study should be an integral part of the quality engineering function. The results could be used for new design applications, inspection planning and evaluation techniques. It is a type of tool that can be used to prevent defects during the production cycle through better designs, through factual knowledge of machine or process limitations and through knowledge of process factors that can or cannot be controlled. In any manufacturing operation there is a variability which is manifested in the product made by the operations .Quantifying the variability with objectives and advantages of reducing it in the manufacturing process is the prime activity of the process management.

Process Capability refers to the evaluation of how well a process meets specifications or the ability of the process to produce parts that conform to engineering specifications, Process Control refers to the evaluation of process stability over time or the ability of the process to maintain a state of good statistical control. These are two separate but

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vitally important issues that we must address when considering the performance of a process and so the assessment of process capability is inappropriate and statistically invalid to assess with respect to conformance to specifications without being reasonably assured of having good statistical control. Before evaluating the process capability, the process must be shown under statistical control i.e. the process must be operating under the influence of only chance causes of variation and also ensure that the process data is normally distributed and observations are independent.

A process may produce a large number of pieces that do not meet the specifications, even though the process itself is in a state of statistical control (i.e., all the points on the X-bar and R charts are within the 3 sigma limits and vary in random manner).This may be due to the lack of centering of the process mean in other words, the actual mean value of the parts being produced may be significantly different from the specified nominal value of the part. If this is the case, an adjustment of the machine to move the mean closer to the nominal value may solve the problem. Another possible reason for lack of conformance to specifications is that a statistically stable process may be producing parts with an unacceptably high level of common-cause variation, even though the process is centered at the nominal value.

2. The basic capability indices commonly used in manufacturing industries are Cp, Cpk Cpm and Cpmk.

Cp: It simply relates the Process Capability to the Specification Range and it does not relate the location of the process with respect to the specifications. Values of Cp exceeding 1.33 indicate that the process is adequate to meet the specifications. Values of Cp between 1.33 and 1.00 indicate that the process is adequate to meet specifications but require close control. Values of Cp below 1.00 indicate the process is not capable of meeting specifications. If the process is centered within the specifications and is approximately "normal" then Cp = 1.00 results in a fraction nonconforming of 0.27%. It is also known as process potential.

Cpl: It estimates process capability for specifications that consist of a lower limit only (for example, strength) and it assumes process output is approximately normally distributed. Cpu: It estimates process capability for specifications that consist of an upper limit only (for example, concentration). Assumes process output is approximately normally distributed.

Cpk: It considers process average and evaluates the process spread with respect to where the process is actually located. The magnitude of C_{pk} relative to C_p is a direct measurement of how off-centre the process is operating. It assumes process output is approximately normally distributed. If the characteristic or process variation is centered between its specification limits, the calculated value for C_{PK} is equal to the calculated value for C_P. But as soon as the process variation moves off the specification center, it is penalized in proportion to how far it's offset. C_{PK} is very useful and very widely used. Generally, a C_{PK} greater than 1.33 indicates that a process is capable in the short term. Values less than 1.33 tells that the variation is either too wide compared to the specification or that the location of the variation is offset from the center of the specification. It may be a combination of both width and location. Cpk measures how far the process mean is from the nearer specification limit in terms of 3σ distances. Cpk works well only for the bell-shaped "normal" (Gaussian) distribution. For others it is an approximation. Cpk = Cp only when the process is perfectly centered. Cp represents the highest possible value for Cpk.

Cpm: It estimates process capability around a target T, it is always greater than zero and assumes process output is approximately normally distributed. It is also known as the Taguchi capability index, introduced in 1988. Cpk measures how well the process mean is centered within the specification limits, and what percentage of product will be within specification limits. Instead of focusing on specification limits, Cpm focuses on how well the process mean corresponds to the process target, which may or may not be midway between the specification limits. Cpm is motivated by Taguchi's "Loss Function". The denominator of Cpm includes the Root Mean Square deviation from the target. Cpk is preferred to Cp because it measures both process location and process standard deviation. Cpm is often preferred to Cpk because the variability term used in the index is more consistent with Run to Target Philosophy.

Cpmk: It estimates process capability around a target (T), and accounts for an off-center process mean and assumes process output is approximately normally distributed. The process capability index - C_{pk} considers process average and evaluates half the process spread with respect to where the process average is actually located, though C_{pk} takes the process mean into consideration but it fails to differentiate an on-target process from off-target process. The way to address this difficulty is to use a process capability index Cpm that is better indicator of centering.

Summary of Process Capability Indices and their usage is presented in the Table 1.

Table 1. List of process capability indices equations and their usage

Index	Estimation Equation	Usage
Cp	$C_p = \frac{(USL-LSL)}{6\,\sigma'}$	It estimates what the process is capable of producing if the process mean were to be centered between the specification limits. Assumes process output is approximately normally distributed.
Cpl	$Cpl = \frac{\bar{\bar{X}}-LSL}{3\sigma'}$ It	It estimates process capability for specifications that consist of a lower limit only. Assumes process output is approximately normally distributed.
Cpu	$Cpu = \frac{USL-\bar{\bar{X}}}{3\sigma'}$	It estimates process capability for specifications that consist of an upper limit only. Assumes process output is approximately normally distributed.
Cpk	$Cpk = Min\,[\frac{USL-\bar{\bar{X}}}{3\sigma'}, \frac{\bar{\bar{X}}-LSL}{3\sigma'}]$	It estimates what the process is capable of producing, considering that the process mean may not be centered between the specifications limits.
Cpm	$Cpm = \frac{Cp}{\sqrt{1+(\frac{\bar{\bar{X}}-T}{\sigma'})^2}}$	It estimates process capability around a target T is always greater than zero. It assumes process output is approximately normally distributed. Cpm is also known as the Taguchi capability index.

Cpmk	$$\text{Cpmk} = \frac{\text{Cpk}}{\sqrt{1 + (\frac{\bar{\bar{X}} - T}{\sigma'})^2}}$$	It estimates process capability around a target T and accounts for an off-center process mean. Assumes process output is approximately normally distributed.

3. Methodology

Estimation of Process Capability for boring operation involves the following steps:

- Understanding the basic concepts of process capability analysis and its measures.
- Process data collection.
- Calculate required statistics
- Validate the critical assumptions.
- Estimation of Cp, Cpu, Cpl, Cpk, Cpm and Cpmk.
- Analysis of process capability results.
- If the process is not capable of meeting the specification, find the predominant factor that affecting the process capability.
- Take action to improve the process performance.
- Estimate the confidence intervals and Carryout Hypothesis Testing.

4. Data collection

Critical quality characteristic of the gear i.e. Bore diameter on the driver gear processed by boring operation in an automotive industry has been identified.The product description is given in the Table 2 and the measured values are presented in the Table 3.

Table 2. Product description

Material: Cast steel	Part Name : Driver Gear
Operation: Boring	Specifications: 205.00 ± 0.05
Instrument used : Dial Bore Gauge	All dimensions are in “ mm”

Table 3. The measured values of Bore diameter

Sample No.	1	2	3	4	5	$\bar{X}$	R
1.	205.030	205.020	205.010	205.045	205.010	205.023	0.035
2.	205.010	205.020	205.025	205.030	205.010	205.019	0.020
3.	205.010	205.030	205.050	205.030	205.020	205.028	0.04
4.	205.030	205.020	205.030	205.040	205.035	205.031	0.02
5.	205.040	205.035	205.030	205.030	205.035	205.034	0.01
6.	205.030	205.030	205.025	205.030	205.035	205.030	0.01
7.	205.025	205.025	205.025	205.025	205.025	205.025	0.00
8.	205.015	205.020	205.025	205.010	205.020	205.018	0.015
9.	205.025	205.030	205.040	205.010	205.010	205.023	0.03
10.	205.025	205.025	205.020	205.010	205.020	205.020	0.015
11.	205.010	205.005	205.030	205.040	205.040	205.025	0.035
12.	205.030	205.020	205.030	205.030	205.030	205.028	0.01
13.	205.030	205.040	205.030	205.040	205.030	205.034	0.01
14.	205.030	205.040	205.030	205.030	205.025	205.031	0.015

15.	205.010	205.010	205.020	205.040	205.050	205.026	0.04
16.	205.035	205.040	205.037	205.042	205.040	205.038	0.007
17.	205.045	205.038	205.045	205.033	205.030	205.038	0.015
18.	205.040	205.030	205.025	205.025	205.020	205.028	0.02
19.	205.030	205.025	205.030	205.030	205.035	205.030	0.01
20.	205.040	205.035	205.030	205.030	205.020	205.031	0.02

5. Process Capability Analysis

According to Kotz and Montgomery (2000) the following critical assumptions have been made and validated before estimating the process capability for boring operation.

- The process must be in state of statistical control.
- The quality characteristic has a normal distribution.
- In the case of two sided specifications, the process mean is centered between the lower and upper specification limits.
- Observations must be random and independent of each other.

All the above assumptions have been verified as follows.

5.1 Construction of $\overline{X}$ and R- Chart to assess the statistical stability of the boring operation

Control limits for $\overline{X}$ - Chart

$UCL = \overline{\overline{X}} + A_2\overline{R} = 205.049 + [(0.577)(0.029)] = 205.06560$,

$LCL = \overline{\overline{X}} - A_2\overline{R} = 205.049 - [(0.577)(0.029)] = 205.03219$

Control limits for R-Chart

$UCL = D_4\,\overline{R} = 2.114 \times 0.029 = 0.06124$

$LCL = D_3\,\overline{R} = 0.00 \times 0.029 = 0.0000$

From Table, for n=5, $A_2 = 0.577$. $d_2 = 2.326$. $D_3 = 0.00$. $D_4 = 2.114$

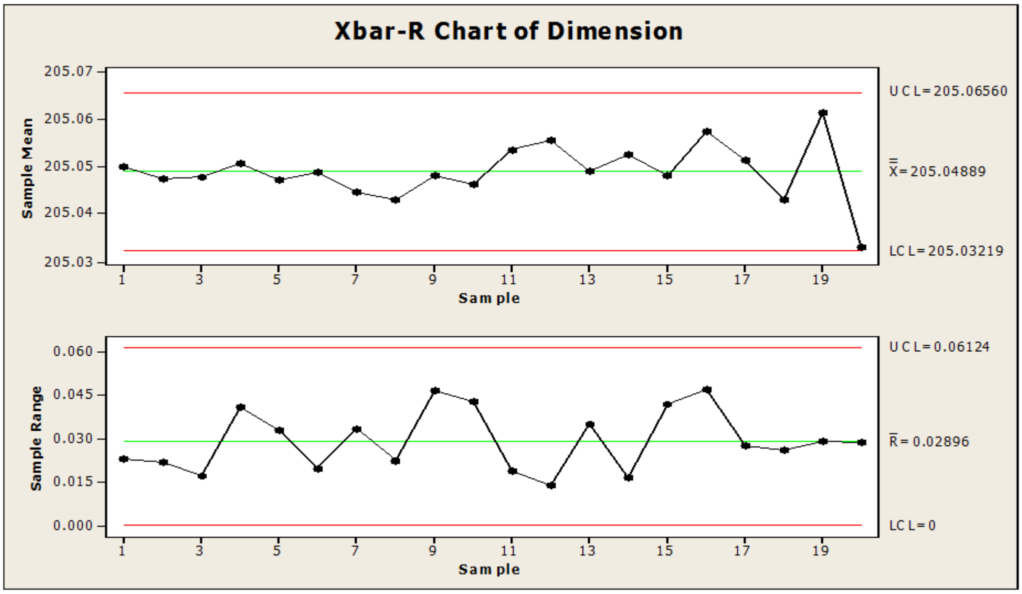

Figure 1. Control charts for case study data

It has been observed from the Figure 1 that all plotted sample range and mean values are within the control limits on both R-Chart as well as X-Bar chart and no indication of Trend, shift, run and clustering has been noticed. Hence, it is concluded that the process is under statistical control and operating under the influence of only chance causes of variation. i.e., the process is stable over time.

5.2 Normal probability plot and histogram for validating the Normality assumption

Graphical methods including the histogram and normal probability plot are used to check the normality of the case study data. Figure 2 displays the histogram and Figure 3 display the normal probability plot for the case study data. The sample data appears to be normal.

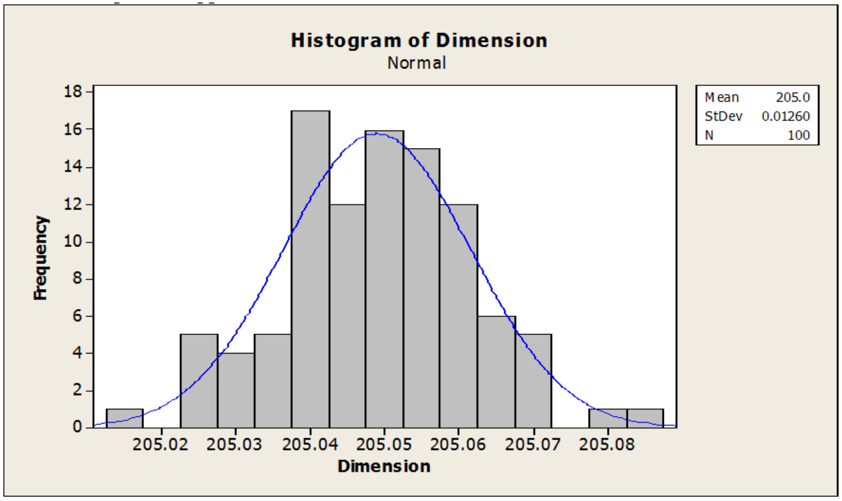

Figure 2. Histogram for the case study data

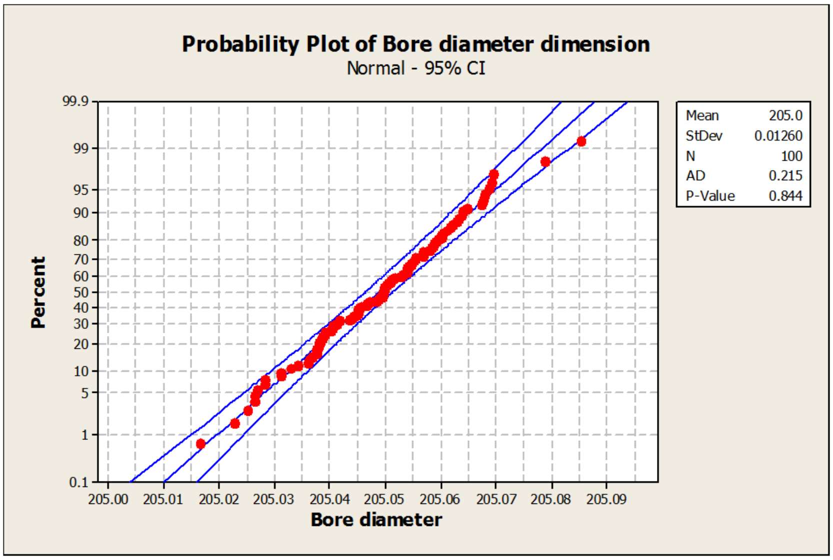

Figure 3. Normal Probability Plot for the case study data

Test results for normal probability plot for the data from MINITAB -14 statistical software output shows Mean: 205.00, Standard deviation: 0.0126, Anderson Darling test statistic value: 0.215 and P-value: 0.844 is greater than the significance level (α = 0.05) implies that the data is distributed normally .Thus, it is concluded that the sample data can be regarded as taken from a normal process.

5.3 Construction of Run chart for checking the assumption of Randomness using MINITAB software

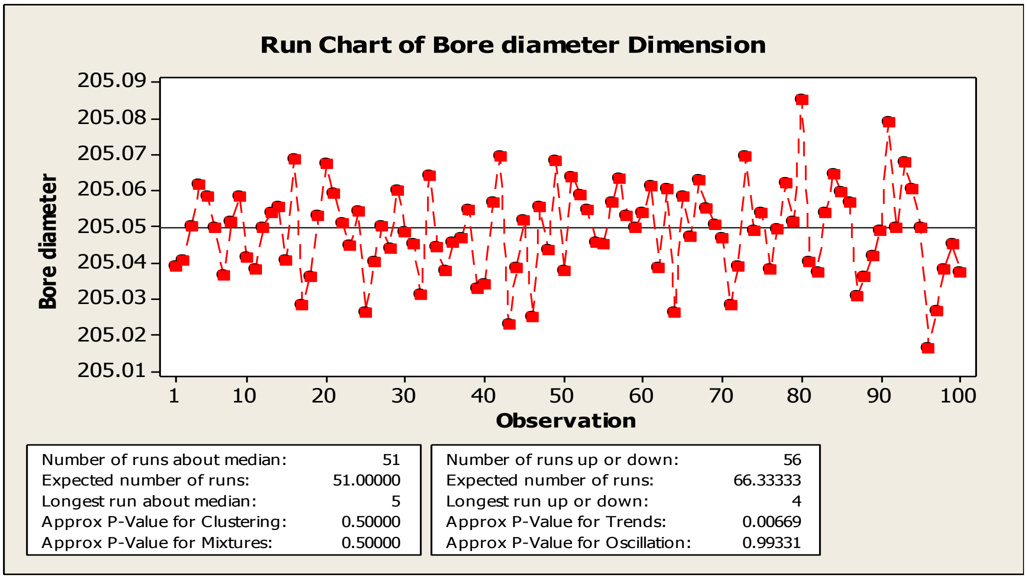

Figure 4. Run chart for case study data

Interpreting the Run Chart: p - values for clustering, mixtures and oscillation are greater than α value of 0.05. In case of the trend, P-value is smaller than the alpha value and it warns that the process is about to go out of control due to the factor like worn tool. The actual numbers of runs are close to the expected number of runs. Hence it is concluded that the observations in the data set are random. After validating the three critical assumptions, the process capability for the boring operation would be quantified.

6. Estimation of Process Capability Indices for existing process conditions

6.1Process Capability Index - C_p

The Average of Averages of the samples ($\bar{\bar{X}}$ = μ) and Average Range ($\bar{R}$) value are computed. The process standard deviation is obtained with the help of formula, σ' = $\bar{R}$ / d_2, value of d_2 for a sample size of five is noted as 2.326 from statistical tables for control chart constants.

The process standard deviation (σ') = $\bar{R}$ / d_2. σ' = 0.029 / 2.326 = 0.01246.

Process capability index,

$$C_p = \frac{(USL-LSL)}{6\,\sigma'} = \frac{(205.050-204.950)}{(6 \times 0.0124)} = 1.34$$

Percentage of specification band used by the current process.

$C_r = (1 / C_p) \times 100 = (1/ 1.34) \times 100 = 74.62$ %.This means that the manufacturing process uses 74.62 % of specification band.

The capability Ratio

$$(CR) = \frac{6\sigma'}{USL-LSL} = \frac{6 \times 0.0124}{205.050-204.950} = 0.744$$

At present the specification range used by the process is 74.4%.

6.2 Process capability index – Cpk (Second- generation capability index, developed from the original Cp)

$$Cpk = \text{Min}\left(\frac{USL-\bar{\bar{x}}}{3\sigma'}, \frac{\bar{\bar{x}}-LSL}{3\sigma'}\right)$$

$$Cpk = \text{Min}(Cpu, Cpk)$$

$$Cpk = \text{Min}\left(\frac{205.050-205.001}{3X\,0.0124}, \frac{205.001-204.950}{3\,X\,0.0124}\right)$$

$$Cpk = \text{Min}(Cpu = 0.026, Cpl = 2.66)$$

Therefore, Cpk = 0.026

6.3. Process capability index- Cpm (Second- generation capability index, developed from the original Cp)

$$Cpm = \frac{USL-LSL}{6\sqrt{\sigma^2+(\bar{\bar{X}}-T)^2}} = \frac{205.050-204.950}{0.3030} = 0.33$$

Where, USL and LSL are upper and lower specification limits.σ' is process standard deviation, $\bar{\bar{X}}$ is process mean, T is target value.

6.4 Cpmk Process capability index-Cpmk (a third- generation capability index that incorporates the features of Cpk and Cpm).

$$Cpmk = \frac{Cpk}{\sqrt{1+\left(\frac{\bar{\bar{X}}-T}{\sigma'}\right)^2}}, \quad \frac{0.026}{4.0761} = 0.0063$$

The case study analysis reveals that C_p is not equal to C_{pk} which implies that the process is not exactly centered. Also, Cp, Cpk Cpm and Cpkm are not very nearer in their magnitude and hence it can be stated that process under study is not exactly centered. It is noticed that even process is under statistical control ,stable over time and have potential to meet the given specification limits, there has been rejections as large as 4,64,626.00 products out of 1 million products due to the shift of the process mean towards upper specification limit as shown in figure 9. In order to reduce the scrap, it is necessary to shift the process mean as close as possible to the target value (i.e., 205.00 mm).

7. Process capability evaluation after shifting the process mean to the specification mean

After adjusting the process mean to the target value of 205.00, data was collected and presented in the Table 4.

Table 4. Measured values of bore dia after adjusting the process mean

Sample	1	2	3	4	5	Range	Mean
1.	205.007	205.006	204.982	205.005	204.996	0.025	204.999
2.	204.995	205.000	205.010	205.021	205.016	0.026	205.008
3.	204.994	205.013	204.990	205.005	204.985	0.028	204.997
4.	204.998	205.016	205.037	204.994	204.991	0.046	205.007

5.	205.019	204.994	205.022	204.983	205.011	0.039	205.006
6.	204.993	205.038	205.028	205.007	204.985	0.053	205.010
7.	205.005	205.015	205.019	205.010	205.026	0.021	205.015
8.	204.985	204.983	205.027	205.005	205.019	0.044	205.004
9.	204.982	204.985	204.982	204.990	205.024	0.042	204.993
10.	204.997	204.987	204.997	204.994	204.973	0.024	204.990
11.	204.986	205.020	204.983	204.993	205.000	0.037	204.996
12.	204.998	205.004	205.007	205.011	205.001	0.013	205.004
13.	204.983	205.017	205.001	204.995	204.985	0.034	204.996
14.	204.986	205.012	205.006	205.014	204.996	0.028	205.003
15.	205.014	204.982	204.998	205.020	205.007	0.038	205.004
16.	205.001	205.013	204.994	205.001	204.980	0.033	204.998
17.	205.020	205.009	204.993	204.995	205.010	0.027	205.005
18.	205.000	205.013	205.001	205.003	205.005	0.013	205.004
19.	204.991	205.017	204.996	204.963	204.992	0.054	204.992
20.	205.015	204.988	205.006	204.980	204.992	0.035	204.996

$\Sigma R=0.66, \overline{R}=0.033, \Sigma \bar{X} = 4100.03, \bar{\bar{X}}=205.001$

8. Process capability evaluation after adjusting the process mean

8.1 Construction of $\overline{X}$ and R- Chart to assess the stability and uniformity of the process

Control limits for $\overline{X}$ –Chart

$UCL = \bar{\bar{X}} + A_2\overline{R}$ = 205.001 + [(0.577) (0.033)] =205.020

$LCL = \bar{\bar{X}} - A_2\overline{R}$ = 205.001 - [(0.577) (0.033)] =204.982

Control limits for R-Chart

UCL= $D_4 \overline{R}$= 2.114 x0.033 =0.070,
LCL= $D_3 \overline{R}$ = 0.00 x 0.033 =0.0000
From table, n=5 $A_2 = 0.577$. $d_2 = 2.326$. D_3 =0.00. D_4 =2.114

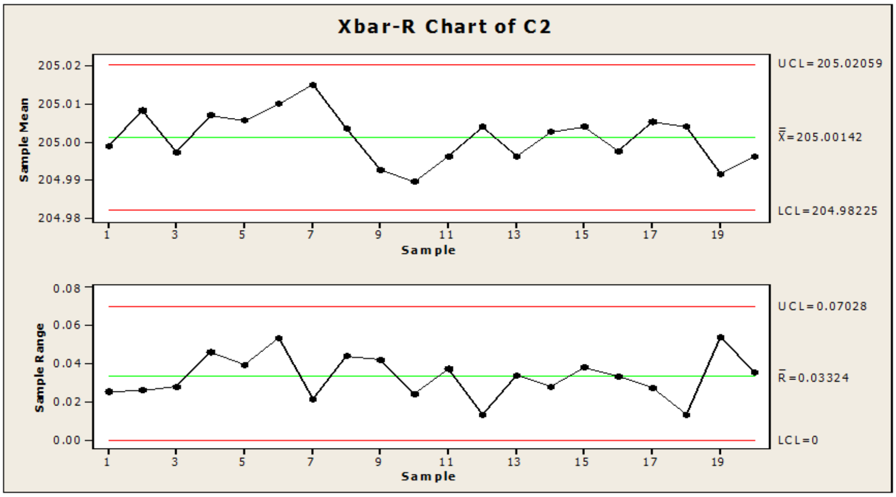

Figure 5. Control charts for case study data

It has been observed that all the plotted sample range and mean values are within the control limits on both R-Chart as well as X-Bar chart and there are no indications of Trend, shift, run and clustering. Hence it is concluded that the process is under statistical control and operating under the influence of only chance causes of variation. i.e., the process is stable over time.

8.2 Normal probability plot and histogram for validating the Normality assumption (After adjusting the process mean).

Graphical methods including the histogram and normal probability plot have been constructed to check the normality of the case study data. Figure 6 display the histogram and Figure 7 shows normal probability plot for the case study data. The sample data appears to be normal.

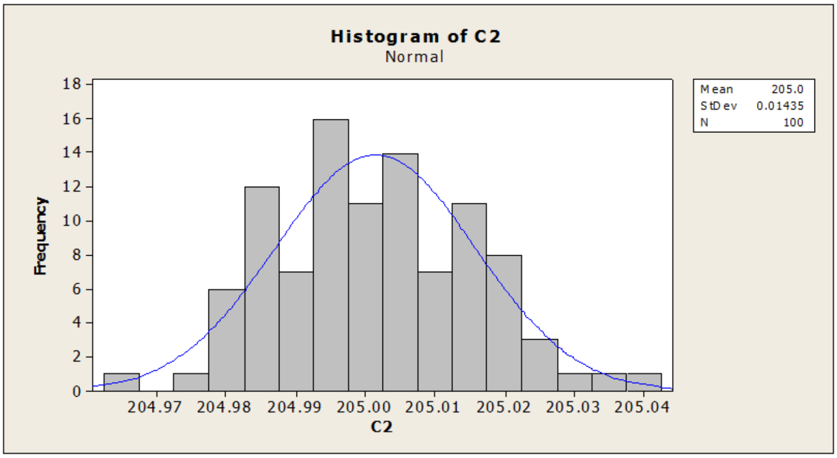

Figure 6. Histogram for case study data

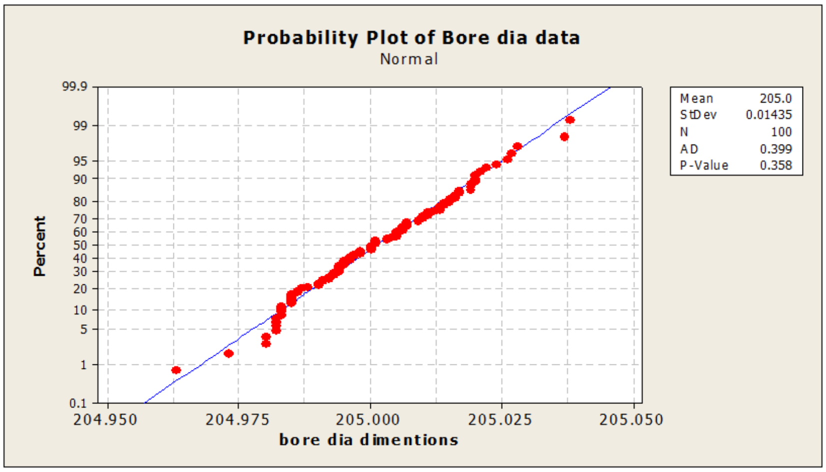

Figure 7. Normal Probabilities Plot for case study data

Test results for normal probability plot for the data from MINITAB -14 statistical software output shows Mean: 205.00, Standard deviation: 0.0143, Anderson Darling test statistic. 0.399 and P- value: 0.358 is greater than the significance level (α= 0.05).This implies that the data is distributed normally .Thus, It has been concluded that the sample data can be regarded as taken from a normal process.

8.3 Construction of Run Chart for checking the assumption of randomness of the case study data

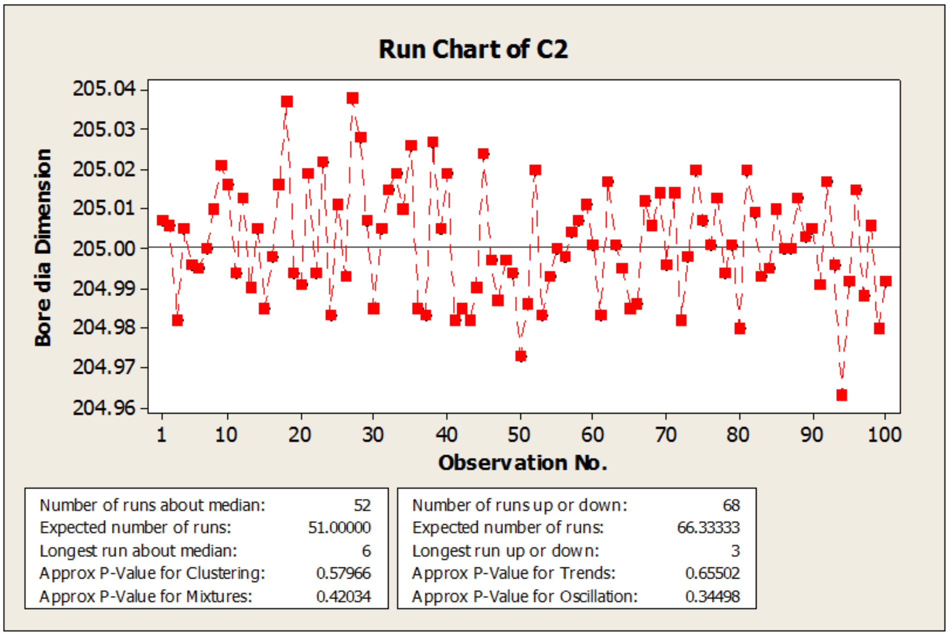

Figure 8. Run chart for case study data

Interpretation of Run chart: The P-values for clustering, mixtures, trends and oscillation are greater than alpha value of 0.05. The actual numbers of runs are close to the expected number of runs. Hence, it is concluded that the data is independent and random.

After validating the three critical assumptions, the process capability of the boring operation would be quantified.

8.4 Process Capability Index - C_p

The mean of the sample means ($\bar{\bar{X}}$ = μ) and average range ($\bar{R}$) value are computed. The process standard deviation is obtained from the formula, $\sigma' = \bar{R} / d_2$. With the help of statistical table the value of d_2 for a sub-sample size of five is noted as 2.326. The process standard deviation is estimated by:

$\sigma' = \bar{R} / d_2$, $\sigma' = 0.033/2.326 = 0.0141$

Process capability index,

$$C_p = \frac{(USL-LSL)}{6\sigma'} = \frac{(205.050-204.950)}{(6 \times 0.0141)} = 1.182$$

Percentage of specification band used by the process $C_r = (1/C_p) \times 100 = (1/1.182) \times 100 = 84.60\%$. This means that the manufacturing process uses 84.60% of specification band.

The capability Ratio,

$$(CR) = \frac{6\sigma'}{USL-LSL} = \frac{6 \times 0.0141}{205.050-204.950} = 0.846$$

The specification range used by the process is 84.46%.

8.5 Process capability index - Cpk (Second-generation capability index, developed from the original Cp)

$$Cpk = \text{Min}\left(\frac{USL-\bar{\bar{x}}}{3\sigma'}, \frac{\bar{\bar{x}}-LSL}{3\sigma'}\right)$$

$$Cpk = \text{Min}(Cpu, Cpk)$$

$$Cpk = \text{Min}\left(\frac{205.050-205.001}{3X\, 0.014}, \frac{205.001-204.950}{3\, X\, 0.014}\right)$$

$$Cpk = \text{Min}(Cpu = 1.158, Cpl = 1.205)$$

Therefore, Cpk = 1.158

8.6 Process capability index - Cpm (Second-generation capability index, developed from the original Cp)

$$Cpm = \frac{USL-LSL}{6\sqrt{\sigma^2 + \left(\bar{\bar{X}} - T\right)^2}} = \frac{205.050-204.950}{0.0848} = 1.18$$

Process capability index-Cpmk (a third-generation capability index that incorporates the features of Cpk and Cpm).

$$Cpmk = \frac{Cpk}{\sqrt{1+\left(\frac{\bar{\bar{X}}-T}{\sigma'}\right)^2}},$$

$$Cpmk = \frac{1.158}{1.0025} = 1.155$$

9. Estimation of non-conforming Gears

9.1 Estimation of non- conforming gears (Before shifting the process mean)

MINITAB-14 Statistical Software has been used to perform process capability analysis and found the number of gears fail to conform to the specification limits per million, as shown in the figure 9.

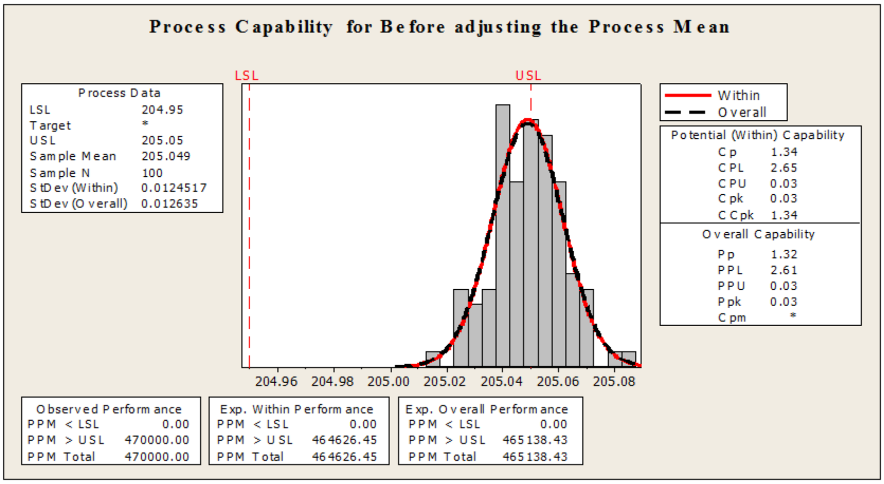

Figure 9. Process Capability Analysis before adjusting the process mean

9.2 Estimation of non-conforming Gears (After adjusting the process Mean to specification Mean)

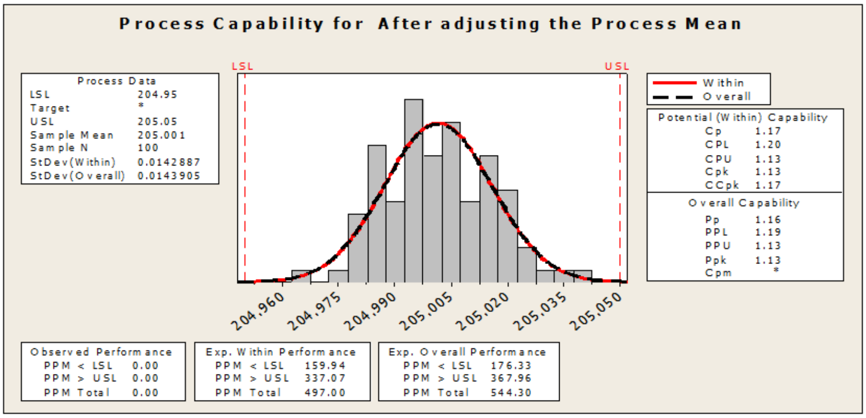

Figure 10. Process Capability Analysis after adjusting the process mean

After adjusting the process mean, it has been noticed that the process is under statistical control, stable over time and capable of meeting the given specification limits. Even after shifting the process mean, it has been noticed from the Figure.10, that rejections as few as 159 gears as scrap and 337 gears as rework out of 1 million gears. Still there is an opportunity to reduce the scrap and rework by identifying and reducing the causes of variation and achieve Cp and Cpk value is 1.33.

10. Estimation of non-conforming Gears

10.1 Estimation of confidence interval for C_p

The expressions for the above capability indices involve population parameters. In practice, they are replaced by their sample estimations (σ' and $\overline{\overline{X}}$), leading to point estimates Ĉp, Ĉpk, Ĉpm and Ĉpmk .Here, confidence intervals have been provided for Ĉp, and Ĉpk under the assumption of normality of the distribution of the quality characteristic. A 100 (1-α)% confidence interval for $\hat{C}p$ (Kushler and Hurley, 1992).

$$\hat{C}p\sqrt{\frac{x^2_{1-\frac{\alpha}{2},n-1}}{n-1}} \leq Cp \leq \hat{C}p\sqrt{\frac{x^2_{\frac{\alpha}{2},\ n-1}}{n-1}}$$

Where $x^2_{1-\frac{\alpha}{2},\ n-1}$ and $x^2_{\frac{\alpha}{2},\ n-1}$ are the lower and upper $\frac{\alpha}{2}$ percentage points on the chi-square distribution with (n-1) degree of freedom.

$$1.182\sqrt{\frac{x^2_{0.975,100-1}}{100-1}} \leq Cp \leq 1.182\sqrt{\frac{x^2_{0.025,100-1}}{100-1}},$$

$$1.182\sqrt{\frac{74.22}{99}} \leq Cp \leq 1.182\sqrt{\frac{129.6}{99}},$$

$$1.182\sqrt{\frac{74.22}{99}} \leq Cp \leq 1.182\sqrt{\frac{129.6}{99}},$$

$$1.023 \leq Cp \leq 1.547$$

10.2 Estimation of confidence interval for C_{pk}

An approximate confidence interval at 95% confidence level for Cpk has been estimated under the assumption that the quality characteristic is normally distributed. (Kushler and Hurley, 1992).

$$\hat{C}pk \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{1}{9n}+\frac{\hat{C}pk^2}{2(n-1)}}$$

Where n represents the sample size used to calculate $\hat{C}pk$ and $Z_{\frac{\alpha}{2}}$ represents the standard normal value for a tail area of $\frac{\alpha}{2}$.

$$1.158 \pm 1.96\sqrt{\frac{1}{9(100)}+\frac{(1.158)^2}{2(100-1)}}$$

$$0.984 \leq \hat{C}pk \leq 1.33$$

11. Testing of Hypothesis

11.1 Testing of Hypothesis for Cp

Testing of the hypothesis has been done whether the product would be accepted by the customer, if the Cp Index for this operation exceeds 1.00, at a significance level of 0.05.

Let, H_o: $C_p \leq 1.00$, $H_1 = C_p > 1.00$ with α = 0.05

In this case, Cp=1.182. A one- sided hypothesis test with α= 0.05, at 95% lower confidence limit of Cp is obtained as below.

$$LCL = Cp\sqrt{\frac{x^2_{0.95,100-1}}{100-1}} = 1.182\sqrt{\frac{x^2_{0.95,100-1}}{100-1}} = 1.048$$

The hypothesized value of Cp =1.0 < LCL. It implies that the true value of manufacturing capability Cp is not less than 1.048 with 0.95 level of confidence. Hence, the null hypothesis (Ho) can't be accepted and conclude that the process is capable of meeting the given specification.

11.2 Testing of Hypothesis for Cpk

Testing of the hypothesis has been done whether the product would be accepted by the customer, if the Cpk Index for this operation exceeds 1.00, at a significance level of 0.05.

Let, H_o: $C_{pk} \leq 1.00$, $H_1 = C_{pk} > 1.00$ with $\alpha = 0.05$.

In this case, Cpk=1.158 .A one - sided hypothesis test with α=0.05, at a 95% lower confidence limit (LCL) of Cpk is obtained as

$$LCL=1.158 - 1.645\sqrt{\frac{1}{9(100)} + \frac{(1.158)^2}{2(100-1)}}$$

Lower Confidence Limit (LCL) =1.012

The hypothesized value of $Cpk(1.0)$ < LCL (1.012), It implies that the true value of manufacturing capability Cpk is not less than 1.048 with 0.95 level of confidence. Hence, the null hypothesis (Ho) can be rejected and conclude that the process is capable of meeting the given specification.

12. Results and discussion

After validating the critical assumptions on the process, process capability for (before and after adjusting the process mean) have been quantified using Cp, Cpl, Cpu , Cpk. Cpm and Cpmk indices and presented in the Table-6 , Table-5 shows commonly used capability requirement and the corresponding precision conditions. In this case, before adjusting the processes mean, Cpmk < 1.00. It implies that process was inadequate. After adjusting the process mean 1.00< Cpmk < 1.33; this indicates that the process is marginally capable and caution needs to be taken regarding the process consistency and rigid process control is required using R-Chart. After adjusting the process mean to the target value i.e., 205.00 mm, Process found to be potential as well as capable of meeting the specification and it requires close control as its Cpk value is less than 1.33. The Cp ≥ Cpm ≥ Cpmk indices show their sensitivity in exhibiting the results. The difference in the Cpmk and the Cp value indicates that the process mean is still not exactly centered with the specification limits .Table 5 shows some commonly used capability requirement and the corresponding precision conditions.

Table 5. Commonly used capability requirement and the corresponding precision conditions

Precision condition	Cpmk Values
Inadequate	Cpmk < 1.00
Marginally capable	1.00≤Cpmk< 1.33
Satisfactory	1.33≤Cpmk< 1.67
Excellent	1.67≤Cpmk< 2.00
Super	2.00 ≤ Cpmk

Table 6. Quantified values of Cp, Cpl, Cpu, Cpk. Cpm and Cpmk indices

Fig.no.	Index	Index value before shifting the Process Mean	Index value after shifting the Process Mean
1	Cp	1.34	1.182
2	Cpk	0.026	1.158
3	Cpu	0.026	1.158
4	Cpl	2.66	1.205
5	Cpm	0.33	1.180
6	Cpmk	0.0063	1.155

13. Conclusion

The case study was conducted in an automotive industry and examined using Cp,Cpk, Cpm and Cpmk index, to show the importance of process capability analysis for monitoring and ensuring the products quality to satisfy the customer's requirements. Before quantifying the indices, validation of the three critical assumptions were tested with the help of statistical tools like control charts, histogram and normal probability plot and run chart using the statistical software-Minitb-14.The quantified values presented in the Table 6. Shown their sensitiveness in exhibiting the results. Among all the indices Cpmk does provide more capability assurance with respect to process yield and process loss to the customers than the other two indices Cpk and Cpm. This is a desired goal according to today's modern quality improvement theory, as reduction of process loss (variation from the target) is as important as increasing the process yield (meeting the specification).The construction of confidence interval for Cpk is not straight forward as that of Cp. In this paper Bissel's approach has been used to construct the confidence interval of Cpk, As it is significantly influenced by sample size,sample size of 100 observations is used for the process capability study.

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