International Journal for Quality research
UDK - 657:334.72
Original Scientific Paper (1.01)

FUZZY-DISTANCE FUNCTION APPROACH FOR MULTIPLE CRITERIA DECISION MAKING

Mayank Kumar [1)]
Prasun Das[2)]

1) B.E., CSE, Birla institute of Technology(Mesra, Ranchi), India mayankb2k@ gmail.com
2) SQC& OR Division, Indian Statistical Institute, Kolkata 108, India prasun@isical.ac.in

Abstract: *In this paper, a method for decision making using fuzzy integral and distance function is presented. Case studies of multiple-response process with correlated responses are used to illustrate the effective application of the proposed approach. The efficacy of this method is compared with the existing methods of MCDM like TOPSIS and GRA. The proposed method is robust, requires less information and less complex as compared to many existing methods.*
Keywords: *MCDM, Fuzzy Integral, Decision Measure, Alternatives.*

1. INTRODUCTION

Decision analysis looks at the paradigm in which an individual decision maker (or decision group) reflects a choice of action in an uncertain environment. The theory of decision analysis is designed to help the individual make a choice among a set of pre-specified alternatives.

The decision making process relies on information about the alternatives. The quality of information in any decision situation can run the whole range from scientifically-derived hard data to subjective interpretations, from certainty about decision outcomes (deterministic information) to uncertain outcomes represented by probabilities and fuzzy numbers. This diversity in type and quality of information about a decision problem calls for methods and techniques that can assist in information processing. Ultimately, these methods and techniques may lead to better decisions.

The actual decision boils down to selecting "a good choice" from a number of available choices. Each choice represents a decision alternative. In the multi-criteria decision making (MCDM) context, the selection is facilitated by evaluating each choice on the set of criteria.

The criteria must be measurable even if the measurement is performed only at the nominal scale (yes/no; present/absent) and their outcomes must be measured for every decision alternative. Criterion outcomes provide the basis for comparison of choices and consequently facilitate the selection of one, satisfactory choice. The basic structure of decision making can be described in three steps: determining the relevant criteria and response; assigning numerical measures to relative importance of criteria on respective responses; and processing the values to determine the ranking of each response.

Some of the well known Multiple Criteria Decision Making (MCDM) methods are weighted sum model (WSM), weighted product model (WPM), analytic hierarchy process (AHP) and simple additive weighting (SAW). Derringer and Suich [1] defined a desirability function to transform several response variables into a single response. Khuri and Conlon [2] simultaneously optimized various responses using polynomial regression models. They firstly defined a distance function by considering the ideal solution, and then determined the optimal condition by minimizing this function. Logothetis and Haigh [3] demonstrated the use of the multiple regression method and the linear programming approach, to optimize a multi-response process using Taguchi experiments.

One of the most useful methods to determine the best response is Technique for Order Preference by Similarity to Ideal Solutions (TOPSIS). TOPSIS was developed by Yoon and Hwang [4] and is considered as one of the effective MCDM methods. TOPSIS applies the principle that the selected alternative should have the shortest distance from the best solution, but have the longest distance from the worst solution [5].

Soft computing paradigm is quite famous now-a-days for multiple criteria decision making. Su and Hsieh [6] and Tong and Hsieh [7] applied artificial neural networks (ANN) to find the optimal solution to the multi-response type of problem. Das [8] applied a TOPSIS driven MCDM procedure for neural network modeling.

A Fuzzy MADM procedure was proposed by Tong and Su [9]. Unlike TOPSIS and other similar methods, which are dependent on the type of response i.e. continuous and categorical, or smaller-the-better, larger-the-better and nominal-the-best type response; it has been tried to develop a method which can find the best alternative among a set of alternatives. To do so, a distance function is developed initially which measures the distance of instance from ideal solution; and then Sugeno integral [10] and fuzzy methods are applied to find out the best alternative.

2. PRELIMINARIES

In the following, we introduce some basic concept related to fuzzy sets.

1. Definition 1:

Let X b e a universe of discourse, then a fuzzy set is defined as:

A= $\{\langle x, \mu_A(x)\rangle \mid x \in X\}$

which is characterized by a membership function $\mu_A : X \rightarrow [0, 1]$, where $\mu_A(x)$ denotes the degree of membership of the element '*x'* to the set *A* [11].

2. Definition 2:

For each fuzzy set A in X, if

$\pi_A(x) = 1 - \mu_A, \forall x \in X$

Then $\pi_A(x)$ is called the degree of indeterminacy of *x* to A [12, 13].

3. PROPOSED METHODOLOGY

The proposed decision making procedure is as follows:

3.1 Obtaining the data set:

Suppose $A = [a_1 \; a_2 \; \cdots \; a_n]$ and $O = [o_1 \; o_2 \; \cdots \; o_n]$ are the set of *n* alternatives and the set of *m* objectives characterizing the decision situation. Then the data set involved in multiple-criteria decision making can be expressed by a matrix:

$$R = \begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{array} \begin{array}{c} \begin{matrix} o_1 & o_2 & \cdots & o_m \end{matrix} \\ \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{bmatrix} \end{array}$$

where r_{ij} represents the value of j^{th} criterion for i^{th} alternative, j = 1 to m and i = 1 to n.

3.2 Weight Assignment:

In MCDM, the weights of attributes, representing their relative importance or value trade-offs, are normally determined in accordance with the subjective preference or perception of the decision makers. Quite a few methods for determining attribute weights have been proposed. However, no single method can guarantee accurate result, and the same people may obtain different weights using different methods [14]. In practical applications, this implies that there is no easy way for determining attribute weights and there are no criteria for determining what the true weight is [15]. In our proposed methodology if the weight vector is *W*, then

$W = [w_1 \; w_2 \; \cdots \; w_m]$

such that $\sum_{j=1}^{n} w_j = 1$

3.3 Ideal Solution:

Obtain the ideal solution. Suppose that the ideal solution vector be *I*, then

$I = [s_1 \; s_2 \; \cdots \; s_m]$

The ideal solution should be the theoretically best solution for each criterion or the most satisfactory empirical result or a reference to compare. The ideal solution is provided by the professional/learned user based on the experience or requirement. We will select the alternative closest to the ideal solution. Supposedly, the ideal solution for a smaller-the-better criterion will be some small value of that response, for a larger-the-better type criterion it would be a high value and for a nominal-the-best it would be the target value of that response. Unit of measurement in case of the ideal solution should be the same as in case of original data set.

3.4 Absolute difference:

Determine the absolute difference of each observation from ideal solution. If d_{ij} is the absolute difference of each criterion from the ideal solution, then

$$d_{ij} = |r_{ij} - s_j|$$

In matrix form, it can be represented as,

$$D = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1m} \\ d_{21} & d_{22} & \cdots & d_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ d_{n1} & d_{n2} & \cdots & d_{nm} \end{bmatrix}$$

where r_{ij} represents the value of j^{th} criterion for i^{th} alternative, s_j is the ideal solution for j^{th} criterion; j = 1 to m and i = 1 to n .

3.5 Normalization:

Normalize d_{ij} in range of [0 1] using the formula, for each *i* and *j* as

$$d'_{ij} = \frac{\left(d_{ij} - \min_{j \in m} d_{ij}\right)}{\left(\max_{j \in m} d_{ij} - \min_{j \in m} d_{ij}\right)}$$

If D' is the matrix of normalized values, then

$$D' = \begin{bmatrix} d'_{11} & d'_{12} & \cdot & \cdot & \cdot & \cdot & \cdot & d'_{1m} \\ d'_{21} & d'_{22} & \cdot & \cdot & \cdot & \cdot & \cdot & d'_{2m} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ d'_{n1} & d'_{n2} & \cdot & \cdot & \cdot & \cdot & \cdot & d'_{nm} \end{bmatrix}$$

where $d'_{ij} = \mu_{o_i}(a)$ will be represented as degree of membership of criterion o_i in alternative 'a'.

3.6 Fuzzy decision model:

Step 1:

The decision measure for a particular alternative 'a', can be implemented as a form,

$M(O_i(a), W_i) = \pi_{w_i} \vee O_i(a)$

Justification of the implication as an appropriate measure can be developed using an intuitive argument [16]. Various criteria can have same weight in a cardinal sense, but they will be unique in an ordinal sense even though $w_i = w_j$ for $i \neq j$ can exist for some criterion. Now, given the weight of each criterion W, reasonable decision models will be the joint intersection of r decision measures,

$$D = \cap_{i=1}^{n} \left| \pi_{w_i} \cup o_i \right| \quad \text{............(1)}$$

where π_{w_i} (inverse of weight) = 1 - w_i

The above equation (1) can be represented in membership form as follows [10];

if $C_i = \pi_{w_i} \cup o_i$, then

$$\mu_{C_i} = \max \left[\pi_{w_i}, \mu_{O_i}(a) \right] \quad \text{....(2)}$$

This expression, given by eqn. (2) can be explained in following way;

As the i^{th} criterion becomes important in the final decision measure, w_i increases, causing π_{w_i} (1-w$_i$) to decrease, which in turn causes $C_i(a)$ to decrease, thereby increasing the chances that $C_i(a) = O_i(a)$, where now $O_i(a)$ will be the value of the decision function, D, representing alternative 'a'.

For a particular criterion, the negation of its weight acts as a boundary such that all rating of alternatives below that boundary become equal to the value of that boundary.

Here, we disregard all distinctions less than that boundary while keeping distinctions above the boundary [16].

Step 2:

The optimum solution a^*, is the alternative which maximizes D.

Using the above defined method, the optimum solution, expressed in membership form, is given by

$$\mu_r(a^*) = \max_{a \in A} \left(\min \left\{ \mu_{c_1}(a), \mu_{c_2}(a), \mu_{c_3}(a), \ldots\ldots\ldots \mu_{c_n}(a) \right\} \right) \quad \text{..........(3)}$$

Step 3:

If two alternatives, x and y, are tied i.e. if their respective decision values are found equal, $D(x) = D(y) = \max_{a \in \alpha}[D(a)]$, where a=x=y; Since, $D(a) = \min_i [C_i(a)]$ there exist some criterion such that $C(a) = D(x)$ and some criterion α such that $C_\alpha(a) = D(x)$ and some criterion β such that $C_\beta(a) = D(y)$. Let

$D^*(x) = \min_{i \neq \alpha} [C_i(x)]$ and $D^*(y) = \min_{i \neq \beta} [C_i(y)]$

Then, compare $D^*(x)$ and $D^*(y)$, and if, $D^*(x) > D^*(y)$, select x as optimum alternative. However, if tie still exists, i.e. $D^*(x) = D^*(y)$, then repeat this step till distinct optimal alternative is obtained.

3.7 Ranking:

Rank the alternatives according to the optimal membership values obtained using *Step 2* and *Step 3* of *section 3.6.*

4. IMPLEMENTATION AND RESULTS

We have applied our proposed methodology for solving few problems taken from different domain.

4.1 Problem 1: Customer satisfaction for technical *measure*

In this problem, various customer needs are defined as criteria or responses, and different technical measure as the alternatives [17]. All the criteria are categorical in nature. The customer satisfaction for a particular customer need and a given technical measure are quantified by a number from 1 to 10, where higher number implies better satisfaction. In the defined problem, 9 alternatives and 10 criteria are specified respectively. The aim is to determine the best alternative among the set of alternatives for simultaneous optimization of 10 criteria. The original data set for this problem has been provided in Table 1. Here W1 to W10 are 10 customer needs, while H1 to H9 are 9 technical measures.The ideal solution in this problem is defined as

$I = [9 \quad 9 \quad 7 \quad 7 \quad 9 \quad 9 \quad 7 \quad 9 \quad 9 \quad 7]$

while the weight for each criterion is

$W = [9\ 7\ 7\ 5\ 3\ 7\ 5\ 5\ 3\ 1]$

Table 1. Customer response for 9 technical measures Customer need

	W1	W2	W3	W4	W5	W6	W7	W8	W9	W10
H1	9	7	7	7	3	5	3	5	1	1
H2	7	9	7	7	3	5	3	5	1	1
H3	1	5	3	1	9	3	1	3	1	1
H4	1	5	1	1	9	3	1	3	1	3
H5	5	5	5	5	7	9	7	7	5	3
H6	3	5	1	1	3	3	1	9	7	5
H7	1	5	3	3	3	3	1	7	9	5
H8	7	5	7	5	5	5	3	3	1	7
H9	5	7	5	7	5	5	3	5	1	7

Using our proposed method, we have computed the order of preference to be H1, H2, H8, H9, H5, H7, H6, H3, H4.When compared with TOPSIS result, the order of preference is found as H5, H1, H2, H8, H9, H6, H7, H3, H4. Therefore, the result of our proposed method is almost comparable with that of the TOPSIS.

4.2 Problem 2:

In this problem, 3 alternatives A1 , A2 and A3 among which decision has to be chosen, and, also, 5 benefit criteria C1 ,........., C5 are identified as the evaluation criteria for these alternatives [18]. The dataset is given in Table 2 and corresponding weight vector are specified.

Table 2. Dataset for 3 alternatives

	C1	C2	C3	C4	C5
A1	7.7	7	7.7	9.67	5
A2	8.3	10	9.7	10	9
A3	8	9	9	9	8.3

The ideal solution of this problem is defined as
I = [0.36 0.41 0.41 0.41 0.40]while the weight for each criterion is
W = [0.9 1 0.93 1 0.63] When simulated on the proposed method, the order of preference was obtained to be A3, A2, A1. When the result was compared with the Fuzzy-TOPSIS method as prescribed in [18], it was found to be A2, A3, A1. Therefore, in this case too result obtained from our proposed method was found comparable.

4.3 Problem 3:

An investment company wants to invest a sum of money in the best option [19]. There is a panel with five possible alternatives to invest the money: 1: A1 is a car company; 2: A2 is a food company; 3. A3 is a computer company; 4. A4 is an arms company; 5. A5 is a TV company. The investment company must take a decision according to the following four attributes: 1. G1 is the risk analysis; 2. G2 is the growth analysis; 3. G3 is the social-political impact analysis; 4. G4 is the environmental impact analysis. The five possible alternatives A_i (i=1, 2,.....5) are to be evaluated as follows

$$R = \begin{array}{c|cccc} & G1 & G2 & G3 & G4 \\ \hline A1 & 5 & 6 & 4 & 4 \\ A2 & 4 & 3 & 5 & 7 \\ A3 & 4 & 5 & 2 & 2 \\ A4 & 5 & 4 & 5 & 3 \\ A5 & 1 & 4 & 7 & 7 \end{array}$$

The weight vector defined is W = [0.15 0.35 0.3950 0.1050] while the ideal solution being defined as
I = [0.2500 0.2042 0.2842 0.4575]

Using our proposed method, the order of preference was obtained to be A2, A5, A1, A3, A4.

When compared with TOPSIS-Incomplete weight information method [20] result, the order of preference is found as A1, A3, A2, A4, A5. In this case, the result of our proposed method varies to a large extent.

4.4 Problem 4: Vendor selection

It is assumed that five vendors are able to supply certain raw materials. The delivery record is rearranged by purchasing staff as 1.Quality (Defects), 2.Price (Unit price), 3.Delivery date (Delay rate), 4.Quantity (Shortage rate), 5.Services (Score) shown in Table 4 [21].

Table 4. Measurement value for each evaluation attributes
(Delivery data for a period of two years)

	Quality	*Price*	*Delivery*	*Quantity*	*Service Date*
A	0.15	12	0.15	0.05	2
B	0.22	10	0.25	0.08	4
C	0.15	8	0.15	0.05	5
D	0.08	13	0.30	0.15	4
E	0.12	9	0.05	0.20	3

The weighting value determination can be done by Delphi Method or Eigenvector [22] and is given by
W = [0.30 0.20 0.15 0.15 0.20]
and the ideal solution is given by
I = [1 1 1 1 1]

Using the proposed method, order of preference was obtained to be C, B, E, D, A. When compared with

GRA method [21], the order of preference was found to be C, E, D, A, B. In this case apart from one selection the result was found comparable.

5 DISCUSSIONS

A few assumptions have been made while applying the proposed method. Firstly, in case of obtaining the ideal solution as input from the user, we are assuming that the user has enough experience and data to provide a reasonable ideal solution. Secondly, while measuring a particular response, unit of measurements should be same for each alternative. Also, unit of measurement for ideal solution should be same as it is in original data set.

The method proposed takes on the ideal solution from user for individual criteria and find out the distance of a criterion from respective ideal solution. Then, the concept of fuzzy integral and fuzzy max-min operation is applied to determine the membership of a criterion for an alternative and decision making. The criteria that maximizes the value for an alternative is selected and then ranked accordingly to get the final decision model. The proposed method is suitable for any kind of response. So, if a situation is there to optimize some responses which are continuous in nature while other being categorical in nature, this method can be applied. Responses can be classified into three distinct groups; smaller-the-better, larger- the- better and nominal-the-best. Bu t in our proposed method, the user is not required to mention the type of response. Instead, while providing the ideal solution, response type should be considered for respective responses. Hence, this method gives same treatment to all the three distinct groups of responses.As the method does not depend on independent variables, the best alternative could be obtained without considering their constraints. We only require the different criteria for each of the alternatives and weight of each criterion.

5.1 Comparison with existing methods:

As the method is quite similar to that of basic principle of TOPSIS or Grey Relational Analysis (GRA), pros and cons of the proposed method are discussed to conclude.

- As compared with TOPSIS, the proposed method also uses the concept of ideal solution. In case of TOPSIS, the ideal solution is obtained from the data set only while in the proposed method, the user are free to use and choose the ideal solution as per the requirement based on domain knowledge. However, the major disadvantage is that, since we are providing the ideal solution of the responses (criterion) instead of validating, user having constrained knowledge of the domain may be prone to make liable error.
- The proposed method is free from the concept of anti-ideal solution unlike TOPSIS.
- Unlike TOPSIS which is limited to input constraints, the proposed method has advantage over it.
- GRA implements different normalization technique for smaller-the-better, larger-the-better and nominal-the-best type responses (criterion); while in the proposed method we do not have different normalization technique for different category of responses.

6 CONCLUSIONS

The methodology of decision making for multiple-criteria problem using fuzzy distance function approach is presented in this paper. For finding the best alternative, relative distance is obtained and then fuzzy integral is applied to obtain the membership of a criterion for an alternative. The maximum membership value for an alternative in decision model is then selected. In order to demonstrate the potential of proposed methodology, few case examples are presented. The proposed method has advantage over existing method as it is less complex than the existing methods and it requires less information about the response types than other methods.

REFERENCES:

[1] Derringer G, Suich R (1980) Simultaneous optimization of several response variables. J Qual Technol 12:214 219

[2] Khuri AI, Conlon M (1981) Simultaneous optimization of multiple responses represented by polynomial regression functions. Technometrics 23:363 375

[3] Logothetis N, Haigh A (1988) Characterizing and optimizing multi-response processes by the Taguchi method . Qual Reliab Eng Int 4:159169

[4] Yoon K., Hwang C.L., 1995. Multiple attribute decision making: an introduction. Sage, California

[5] Liang G., 1999, Fuzzy MCDM based on ideal and anti-ideal concepts. European Journal of Operational Research 112, 3, 682-691.

[6] Su CT, Hsieh KL (1998) Applying neural networks to achieve robust design for dynamic quality

characteristic. Int J Qual Reliab Manage 15(45):509 519
[7] Tong LI, Hsieh KL (2000) A novel means of applying artificial neural networks to optimize multi-response problem. Qual Eng 13(1):11 18
[8] Das P., 2010. In search of best alternatives: a TOPSIS driven MCDM procedure for neural network modelling. Neural Comput & App lic 19, 91-102
[9] Tong. L., Su C., 1997. Optimizing multi-response problems in the Taguchi method by fuzzy multiple attribute decision making. Quality and Reliability Engineering International 13, 25- 34.
[10] Grabisch M., Murofushi T., Sugeno M., Fuzzy Measure of Fuzzy Events defined by Fuzzy Integrals, Fuzzy Sets & System 50, 293-313.
[11] Zadeh L. A., Fuzzy sets, Information and Control, Vol. 8, pp.338- 356, 1965
[12] Atanassov K., Intuitionistic fuzzy sets, Fuzzy Sets and Systems, Vol. 20, pp.87-96, 1986.
[13] Atanassov K., More on intuitionistic fuzzy sets, Fuzzy Sets and Systems, Vol.33, pp.37- 46, 1989.
[14] F. H. Barron, B. E. Barrett, Decision quality using ranked attribute weights, Management Science 42(11) (1996) 1515-1523.
[15] M. Weber, K. Borcherding, Behavioral influences on weight judgments in multiattribute decision making, European Journal of Operational Research 67(1993) 112.
[16] Yager R., 1981. A new methodology for ordinal multiobjective decisions based on fuzzy sets. Decision Sci., vol. 12, 589-600.
[17] Wu H., 2002-03, A comparitive study of using Grey relational analysis in multiple attribute decision making problems. Quality Engineering 15, 2, 209-217
[18] Jahanshahlo o G.R., Hosseinzadeh Lot F., Izadikhah M., Extension of the TOPSIS method for decision-making problems with fuzzy data. Elsevier Inc.
[19] Herrera F., Herrera- Viedma E., Linguistic decision analysis: steps for solving decision problems under linguistic information, Fuzzy Sets and systems, vol. 115, no. 10, pp. 67-82, 2000.
[20] Jianli Wei, TOPSIS Method for Multiple Attribute Decision Making with Incomplete Weight Information in Linguistic Setting. Journal of Convergence Information Technology. Volume 5, Number 10. December 2010
[21] Chih-Hung Tsai, Ching-Liang Chang, and Lieh Chen, Applying Grey Relational Analysis to the Vendor Evaluation Model. International Journal of The Computer, The Internet and Management, Vol. 11, No.3, 2003, pp. 45 – 53
[22] Saaty, Y., The Analytic Hierarchy Process, Mcgraw-Hill, New York, 1980.

Received: 31.11.2011 Accepted: 20.03.2012 Open for discussion: 1 Year