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SHORTCUT METHODS FOR SIMPLEX-BASED SENSITIVITY ANALYSIS OF LINEAR PROGRAMMING AND RELATED SOFTWARE ISSUES

Abstract: *This paper has presented an overview of theoretical and methodological issues in simplex based sensitivity analysis (SA). The paper focuses somewhat on developing shortcut methods to perform Linear Programming (L.P.) sensitivity analysis manually and in particular to dual prices and its meaning and changes in the parameter of the L.P model. Shortcut methods for conducting sensitivity analysis have been suggested. To perform sensitivity analysis in real life, one needs computer packages (software) to achieve the sensitivity analysis report for higher accuracy and to save time. Some of these computer packages are very professional, but, unfortunately, some other packages suffer from logical errors in the programming of sensitivity analysis.*

Keywords: *L.P. model, Sensitivity analysis (SA), Simplex, Shadow price, RHS, computer packages*

1. Introduction

Many managerial decisions hinge on the issue of how to make the most of the company's resources of raw material, manpower, time, and facilities. L.P. is a technique that aims at optimizing performance regarding combinations of resources. LP can offer managers the capability of building scenarios through its extensive "what if" analysis and sensitivity analysis facilities. While most practical L.P. problems would require a very long time to solve manually, computer software can be utilized to arrive at a solution in a very short period.

When we are dealing with sensitivity analysis, we are initially looking into changes might happen to the parameters of L.P. model. These possible changes would imply to investigate the changes in RHS of the model constraints and the coefficient of the objective function (Baird, 1990).

Once the optimal solution to an L.P.problem has been achieved using the Simplex algorithm, it may be desirable to study how current optimal solution stays optimal when one or more of the problem parameters may change. It is crucial to figuring out how sensitive the optimal solution is to some changes in the model parameters (Bianchi & Calzolari, 1981). Sensitivity analysis, (post-optimality), therefore, looks at "what if" questions scenarios. What happens to the cash position, for example, if sales fall by 5%? What happens if primary supplier increases raw material prices by 12%?.

When we deal with practical problems, sensitivity analysis is much more important than the result obtained from the optimal

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solution. Such an analysis transforms the L.P. solution into a valuable tool to study the effect of changing conditions such as in management, business, and industry. When we include the organization's business plan with the sensitivity analysis report, it will show that we have thought about some of the potential risks - and that is halfway to avoiding them.

Sensitivity analysis can help in making proper decisions. For example, if we may want to consider, the effect of increased labor force or decrease overhead charges, or reducing capacities, due to over-optimistic forecasts, what effect of these actions on counteracting competitors.

2. Literature Review

The literature on SA is enormous and diverse. In late 1980's and early 1990's several researchers and scientists were involved in the fields of operations research working on the L.P. sensitivity analysis topic. Some significant advances were produced in L.P. sensitivity analysis and related problems.

The research in the field of SA was extensively carried out by many O.R. specialists. It includes Murty (1983), Luenberger (1984), Eschenbach and McKeague (1989), Bazaraa and Jarvis (1990), Hamby (1994), Murty (1995), Clemson (1995), Bradley et al. (1977), Gass S. (1997). Gal and Greenberg (1997) worked on sensitivity analysis parameter but excluded the simultaneous changes in the LP parameter.

Arsham (1992) studied the SA for the parameters of structured problems. Khan et al. (2011) studied the profit in products by using LP techniques and SA.

However, the existing literature concerning our research scope is limited. Most of the previous work on sensitivity analysis were focused on lengthy methodologies and procedures that consume a significant amount of time to arrive at a solution of sensitivity analysis.

Yang (1990) introduced two kinds of SA. First one is defining the properties of sensitivity region while the second one is the positive sensitivity analysis.

Most of the OR software packages available for solving L.P. do resolve not only the L.P. problem but also provide the details on the SA of the optimal solution to certain changes in the data. This method has been presented in many papers and textbooks up to now (Dantzig (1963, 1978); Gal, (1979)).

3. Research objectives

Calculations the ranges for optimality and feasibility in sensitivity analysis are covered in most operations research books and most quantitative textbooks. The methods used vary from one book to another although all of them achieve the same results. These methods may take a very long time and effort to solve the sensitivity issues manually. Many of these methods have the tendency to involve lengthy mathematical approaches that they require students to be well-equipped with advanced mathematical techniques such as matrices and vectors. Some other methods need a longer routine to get the results. It is much practical to use lighter and sounder methods to obtain the sensitivity analysis results with ease and with less time. It is known that in real daily problems, we can use one of the many OR packages to solve these issues rather than using manual methods. The latter is to be adopted in the research as it recommends the use of shortcut methods in academic institutions. Thus, the paper will explain how the shortcut methods can be used to derive the sensitivity analysis results.

4. Shortcut Methods to perform Simplex-Based Sensitivity Analysis of LP Problems

To apply Sensitivity analysis of LP problems, the optimal solution of Simplex method must be available. The essence of the sensitivity analysis is to examine how marginal changes in the parameter of the problem might affect the derived optimal solution.

The most taught topics of sensitivity analysis at academic institutes comprise the following items:

1) Changes in the objective function Coefficients (c_i)
2) Changes in the Right-Hand-Side values of the constraints (b_i)
3) Shadow Price (Dual Price) and the Economic Interpretation.

As mentioned earlier, it is expected that the methods of performing sensitivity analysis taught in educational institutes should be easy to apply and short in procedures. In this paper, the author has developed and implemented simple methods for calculating sensitivity analysis that the author has used in teaching Operations Research courses for his long careers, in education. The demonstrations of these methods are shown in the following Table 1.

Example 1:

Table 1. Demonstrations of the methods

Min Z =1.2 X_1+1.75X_2+2.25X_3+ 2.7X_4					
S.t					
25 X_1	+ 10 X_2	+ 17 X_3	+ 10 X_4	$\geq$	150 (grams of nutrition A)
15 X_1	+ 20 X_2	+35 X_3	+70 X_4	$\geq$	250 (grams of nutrition B)
16 X_1	+ 20 X_2	+ 20 X_3	+ 25 X_4	$\geq$	200 (grams of nutrition C)
		X_3		$\leq$	3 (# Kg from grain-1)
X_1	+ X_2	+ X_3	+ X_4	=	10 (K.G. Bag weight)

X_1 , X_2 ,X_3 ,X_4 $\geq$ 0 (Non-Negativity Constraint)

Applying the Simplex Method (Table 2) to solving this problem, we obtain the following optimal solution:

Table 2. Applying Simplex Method

Basis	C_B	X_1 1.2	X_2 1.75	X_3 2.25	X_4 2.7	S_1	S_2	S_3	S_4	a_5	RHS
X_2	1.75	0	1	0.16	0	0.12	0	0.2	0	6.2	4
S_2	0	0	0	1.33	0	-2.33	1	- 10	0	-203.3	66.67
X_4	2.7	0	0	0.37	1	-0.05	0	-0.2	0	-4.53	2.67
S_4	0	0	0	0	0	0	0	0	1	0	3
X_1	1.2	1	0	0.47	0	-0.07	0	0	0	-0.67	3.33
Z		1.2	1.75	1.85	2.7	-0.014	0	-0.19	0	-2.19	18.2
C-Z		0	0	0.4	0	0.014	0	0.19	0	2.19	

4.1. Changes in the Objective Function Coefficients (c_i)

The methods of finding the range of optimality for the parameters of an objective function (C_i) are in two parts. The first part deals with variables related to the Basic decision variables (X_1, X_2, and X_4), and the second part is related to Non-Basic decision variables (X_3).

4.1.1. Basic-Decision Variables

To calculate the range of basic variable, we need to pick up the non-zero values in a *c-z* row of the optimal solution. Exclude the values under the artificial variable columns, if artificial variables exist, and divide them by the corresponding values in the “Basic” decision variable row that we try to find the ranges. To explain the steps, let us consider the“Basic” variable X_1 range (Table 3):

Table 3. “Basic” variable X_1 range

c-z values	0.40	0.014	0.19
Row values in X_1	0.47	-0.070	0
c-z / X_1	0.85	-0.200	+∞

Then, pick up the <u>least positive</u> and <u>least negative</u> product of the division operations (i.e. 0.85 and -0.2) to form the range of C_1. The range of C_1 can be calculated as follows:

$$1.2-0.2 \leq C_1 \leq 1.2+0.85$$
$$1 \leq C_1 \leq 2.05$$

The C_1 range is indicating that the optimal solution will remain optimal as long as the range of C_1 lies between \$1 and \$2.05. Sometimes, when we divide c-z by row values, we do not get one of the signs (+ or -), in this case, we use infinity value for the missing sign. (e.g. if there is no (+) sign produced in the division process, then we use +∞ to be the upper limit).

Now we can apply the same calculation to X_2 and X_4 as follows (Table 4 and Table 5):

Table 4. Calculation to X_2 and X_4

c-z values	0.4	0.014	0.19
Row values in X_2	0.16	0.12	0.20
c-z / X_2	2.5	0.12	0.95

$$\frac{\boldsymbol{c-z}}{\boldsymbol{cx_2}} = \frac{0.4}{0.16} = \mathbf{2.5}$$
$$\frac{0.014}{0.12} = 0.12$$
$$\frac{0.19}{0.20} = 0.95$$
$$1.75-\propto \leq C_2 \leq 1.75+0.12$$
$$-\propto \leq C_2 \leq 1.87$$

(-∝ is used in above range as there is no (-) sign produced by the division process)

Table 5. Calculation to X_2 and X_4

c-z values	0.4	0.014	0.19
Row values in X_4	0.37	-0.05	-0.2
c-z / X_4	1.8	-0.28	-0.95

$$\frac{\boldsymbol{c-z}}{\boldsymbol{cx_4}} = \frac{0.4}{0.37} = 1.0$$
$$\frac{0.014}{-0.05} = -0.28$$
$$\frac{0.19}{-0.2} = -0.95$$

$$2.75-0.28 \leq CX_4 \leq 2.75+1.08$$
$$2.43 \leq CX_4 \leq 3.83$$

The example below is quoted from a book titled “An Introduction to Management Science” by Anderson et al. (2009) to compare the author 's method with the method used by an international textbook. Consider the calculation of C_1 range in the following example (2):

Example 2:

Max $50X_1 + 40X_2$
s.t.
$3X_1+5X_2 \leq 150$
$X_2 \leq 20$
$8X_1+5X_2 \leq 300$
$X_1,X_2 \geq 0$

The final simplex tableau for this problem is shown in Table 6.

Table 6. The final simplex table

Basis	CB	50x1	40x2	S1	S2	S3	RHS
x2	40	0	1	0.32	0	-0.12	12
S2	0	0	0	-0.32	1	0.12	8
x1	50	1	0	-0.20	0	0.20	30
z		50	40	2.80	0	5.20	1980
c-z		0	0	-2.80	0	-5.20	

To find the range of C_1 according to the book method, we assume that X_1's profit contribution is now 50+k, where k is some number representing a change in X's profit contribution. The final simplex tableau is then given by (Table 7):

Table 7. The final simplex table (Continued)

Basis	C_B	$50X_1$	$40X_2$	S_1	S_2	S_3	RHS
x_2	40	0	1	0.32	0	-0.12	12
S_2	0	0	0	-0.32	1	0.12	8
x_1	50	1	0	-0.20	0	0.20	30
z		50	40	2.80-0.2k	0	5.20+0.2k	1980
c-z		0	0	-2.80+0.2k	0	-5.20-0.2k	

The solution will remain optimal as long as all c-z $\leq$ 0. Therefore, for column S_1 we must have:

$-2.8+0.2k \leq 0 \leftrightarrow k \leq 14$

Similarly, we calculate k for S_3as follows:

$-5.2-0.2k \leq 0 \leftrightarrow k \leq -26$

In other words, the current solution will remain optimal as long as X_1's profit contribution lies in the following range:

$$50-26 \leq C_1 \leq 50+14$$
$$24 \leq C_1 \leq 64$$

The range of C_1according to author method can be shown as follows:

$$\frac{\boldsymbol{c-z}}{\boldsymbol{cx_1}} = \frac{-2.8}{-0.2} = 14 \qquad \frac{-5.2}{0.2} = -26$$
$$50-26 \leq C_1 \leq 50+14$$
$$24 \leq C_1 \leq 64$$

The author's methodology is, of course, much simpler and shorter than the method of the textbook in the example above.

4.1.2. "Non-Basic" Decision Variables

In example (1), X_3, is a non-basic decision variable since it did not appear in the basis of the optimal solution. It is "An Over-Priced Good" as we deal with minimizing cost model (it is called under-priced-good in maximization problems). In sensitivity analysis practice, we can find the range of such variable for which it will always be out of the optimal solution. The Variable X_3 is over-priced by \$0.4 (in a c-z row). The price of X_3 should be reduced by at least \$0.4 from its original coefficient (cost) in the objective function to enter the solution and become competitive with the rest of the problem variables. However, in the sensitivity analysis, we are always keen to keep the "non-basic" variable X_3 outside of the Basis column of Simplex table. Therefore, X_3 coefficient should have a range to be kept out of the basis of the Simplex table. The author's method for finding non-basic variable can be presented as follows:

(Original X_3 coefficient- (under-price amount)) < C_3 < (Original X_3 Coefficient $+\propto$)

$$2.25-0.4 < C_3 \leq 2.25+ \propto$$
$$1.85 < C_3 \leq +\propto$$

In general, to determine the range of the coefficient of the non-basic decision variable in a *maximization* problem, we set the upper limit to less than the value of this variable's column in Z row. While we set the lower limit to greater or equal $-\propto$. The "non-basic" decision variable for a *minimization* problem, behaves differently. The upper limit always less than $+\propto$, and "greater than" the value of this variable's column in Z row as the lower limit.

Max Obj. Function $-\propto \leq C_3 <$ Value of X_3 in Z

Min Obj. Function Value of X_3 in Z $< C_3 \leq +\propto$

4.2. Changes on RHS of the constraints (b_i)

The proposed b_i range method developed and implemented by the author over a relatively long period is a very simple and straightforward shortcut method compared to methods that are used in many international textbooks.

To find the range of feasibility of a particular constraint's RHS value (b_i), we perform the following calculations. Divide the RHS column values of the Simplex optimal solution by the corresponding values in the column of the required additional variable associated with the constraint (Slack, Surplus, or artificial variable). For example, if the calculation of b_1 is required, for the first constraint, we divide the values of RHS column in the optimal solution by the values of a ($-S_1$) column. The negative sign that precedes S_i in the denominator of the fraction is s et as default in all calculations of any "b_i" range. We should be aware of bringing S_i also with its sign depending on the type of the constraint ($\leq$, $\geq$, or =). For example, if we try to find the 1st constraint range of feasibility (b_1) in the above example, the denominator of the fraction sign should be ($-(-S_1) = +S_1$).

Referring to the example (1), that is presented earlier in this paper, calculations of "b_i" ranges performed as follows:

$$\frac{\text{RHS}}{-(-S_1)} = \frac{4}{0.12} = 33.33 \qquad \frac{66.67}{-2.33} = -28.6 \qquad \frac{2.67}{-0.05} = -53.4 \qquad \frac{3}{0} = \infty \qquad \frac{3.33}{-0.07} = -47.6$$

$$\frac{\text{RHS}}{-(-S_2)} = \frac{4}{0} = \propto \qquad \frac{66.67}{1} = 66.67 \qquad \frac{2.67}{0} = \propto \qquad \frac{3}{0} = \propto \qquad \frac{3.33}{0} = \propto$$

$$\frac{\text{RHS}}{-(-S_3)} = \frac{4}{0.2} = 20 \qquad \frac{66.67}{-10} = -6.67 \qquad \frac{2.67}{-0.2} = -13.35 \qquad \frac{3}{0} = \propto \qquad \frac{3.33}{0} = \propto$$

$$\frac{\text{RHS}}{-(+S_4)} = \frac{4}{-0} = -\propto \qquad \frac{66.67}{-0} = -\propto \qquad \frac{2.67}{-0} = -\propto \qquad \frac{3}{-1} = -3 \qquad \frac{3.33}{-0} = -\propto$$

$$\frac{\text{RHS}}{-(+a_5)} = \frac{4}{-6.2} = -0.65 \qquad \frac{66.67}{203.3} = 0.33 \qquad \frac{2.67}{4.53} = 0.59 \qquad \frac{3}{-0} = -\infty \qquad \frac{3.33}{0.67} = 4.97$$

The calculated feasibility ranges are:

$$(150-28.6)\ 121.4 \leq b_1 \leq 183.3\ (150+33.3)$$
$$(250-\propto)\ -\propto \leq b_2 \leq 316.67\ (250+66.67)$$
$$(200-6.67)\ 193.33 \leq b_3 \leq 220\ (200+20)$$
$$0 \leq b_4 \leq +\propto$$
$$9.35 \leq b_5 \leq 10.33$$

Most well-known international text books use the much longer method on feasibility ranging of RHS. To show the simplicity and practicality of the suggested method of finding the range of feasibility (b_i), consider the following example.

An example from Hamdi Taha's book (Taha, 2010) given below to provide some comparisons between the two methods:

Suppose we have the following LP model:

Max Z = $3X_1+2X_2+5X_3$ s.t:
$X_1+2X_2+X_3 \le 430$
$3X_2+2X_3 \le 460$
$X_1+4X_2 \le 420$

$X_1, X_2, X_3 \ge 0$

The associated optimum solution for the above problem is (Table 8):

Table 8. Optimum solution

Basic	X_1	X_2	X_3	S_1	S_2	S_3	RHS
z	4	0	0	1	2	0	150
X_2	$-1/4$	1	0	½	$-1/4$	0	100
X_3	$3/2$	0	1	0	½	0	230
S_3	2	0	0	-2	1	1	20

Let us find the range of feasibility for the first constraint (b_1). Hamdi in his book used the following method:

1) He assumes the amount of change in first constraint's RHS=b_1.
2) We need the matrix for the S_i columns in the optimal solution (gray color area in table above)
3) Then we apply the following matrix multiplication as follows:

$$\begin{pmatrix} x_2 \\ x_3 \\ s_2 \end{pmatrix} = \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 0 \end{pmatrix} \begin{pmatrix} 430+b_1 \\ 460 \\ 420 \end{pmatrix} = \begin{pmatrix} 100+b_1/2 \\ 230 \\ 20-2b_1 \end{pmatrix} \ge \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

These conditions lead to the following bounds on D_1:

$$(x_2 \ge 0): 100 + b_1/2 \ \ge 0 \ \rightarrow \ b_1 \ \ge \ -200$$
$$(x_3 \ge 0): x_3 \ is \ independent \ of \ b_1$$
$$(s_3 \ge 0): 20 - 2b_1 \ \ge 0 \ \rightarrow \ b_1 \ \le \ 10$$
$$430 - 200 \le b_1 \le 430 + 10$$
$$230 \le b_1 \le 440$$

Obviously, some skills in matrix multiplication are required, and the associated calculations are time-consuming. To see the difference between Hamdi's methodology, and the author's methodology, let us use the author's method to calculate the range for b_1:

$$\frac{\text{RHS}}{-(+S_1)} = \frac{100}{-1/2} = -200 \qquad \frac{230}{-0} = -\infty \qquad \frac{20}{+2} = 10$$

Select the lowest negative (-200) and least positive (10). The b_3 range is

$$430 - 200 \le b_1 \le 430 + 10$$
$$230 \ \le b_1 \le 440$$

Apparently, we have reached the same answer with much less effort.

4.3. Reduced Cost, Shadow prices, and their Economic Interpretations

For any "non-basic" variable the reduced cost is the amount of improvement needed in the non-basic variable coefficient before it will become a "basic" variable in the optimal solution, while the reduced cost of a basic variable is zero which clearly appears in the net evaluation row (c-z) of the simplex tableau.

If the objective function coefficient of a non-basic variable X_i is improved by exactly its reduced cost, then the LP problem will have alternative optimal solutions. If the coefficient of the non-basic variable X_i improves by more than its reduced cost, the optimal solution will allow the non-basic variable to appear in the basis column of the optimal solution

The shadow price is the amount of improvement in the value of objective function because of change of one unit of resources in particular constraint's RHS.

Sometimes we need to add one unit of resources to the RHS of the i^{th} constraint, then the shadow price for the i^{th} constraint is the amount by which the optimal value of z is "improved." Usually, the Z-value increases in a Maximization problem and decreases in a Minimization problem. A $\geq$ constraint usually produces "non-positive" shadow price while a < constraint will normally have a nonnegative shadow price.

For maximization problems, the less or equal sign constraint ($\leq$) can often be thought of as restrictions on the amount of resources available, and the objective function can reflect the profit. The meaning of shadow price (dual price) is by how much the optimal value of the objective function will either increases or decreases as a result of adding or subtracting one unit of resources in particular RHS of a constraint. The amount of change would give us a good way of interpretation of how much we are willing to pay for an extra unit of resources.

In example (1), at least 150 units are required for the first constraint as the sign is ($\geq$). The shadow price of the first constraint can be determined by quoting the value in row z of the optimal solution tableau under the first constraint surplus variable (S_1) column. The shadow price, or sometimes called Dual price, cannot be computed straight forward from row z as in example one. There are three dimensions connected to the shadow price value. The author has suggested a 100% rule to calculate the shadow price for any LP problem (Maximization or Minimization). Moreover, this rule will apply whether the type of the corresponding constraint is $\leq$, $\geq$ or =. The suggested method for calculating the shadow price is as follows:

Suppose we want to find the shadow price of the first constraint of example 1:

Shadow price for 1st Constraint = [value of additional variable (S_1) in z-row]* [Type of constraint sign (- for $\geq$ sign and + for $\leq$ or = constraint]*[type of objective function (+ for Max., - for Min.)]

Shadow price for 1st Constraint = [-0.014]*[-($\geq$)]*[-(Min)] =-0.014
Shadow price for 2nd Constraint= [0]*[-($\geq$)]*[-(Min)] =0
Shadow price for 3rd Constraint= [-0. 19]*[-($\geq$)]*[-(Min)] =-0.19
Shadow price for 4th Constraint= [0]*[+($\leq$)]*[-(Min)] =0
Shadow price for 5th Constraint= [-2. 19]*[+(=)]*[-(Min)] =+2.19

The economic interpretation of shadow price has a significant meaning in the decision-making process. It shows the actual effect (i.e., increasing or decreasing) in the objective function value as a result of adding one unit of resources or requirement to the RHS of the constraint. The method suggested by the author is expected to help researchers to avoid common errors of interpretations of the shadow prices that are usually caused by the variations in types of LP models as well as in types of constraints.

Economic Interpretation (E.I) of Shadow price= [Shadow price of i^{th} constraint]*[Type of objective function (Max or Min].

By applying this method, shadow price can be said to increase (+)/decrease (-) for profit (Max) or cost (Min). If we apply this rule to example 1 the results will be as follows:

The (E.I) for 1st Constraint Shadow Price, =[-0.014]*[-Min]=+0.014

The interpretation that the cost of the objective function will increase by \$0.014 as a result of adding one gm. of Nutrition A in the mixture.

E.I. for 2nd Constraint shadow price=0

E.I. for 3rdConstraint shadow price= [-0.19]*[-Min] =+0.19

E.I. for 4th Constraint shadow price=0

E.I. for 5th Constraint shadow price= [+2.19]*[-Min] =-2.19

The economic interpretation of the first constraint suggests that if we increase the RHS of the first constraint by just one unit (gram), then the cost of the objective function will increase by \$0.014. It implies that we add an extra gram of Nutrition A in the mixture, the cost of the bag will increase by \$0.014. Similarly, we can interpret other shadow prices.

5. Performing sensitivity analysis using computer software

There are various Operations Research Software (computer packages). Some of these computer packages are usually attached to books of OR and Quantitative Methods. The main purpose of these computer packages is to calculate the optimal solution of OR problems using a variety of suitable OR techniques such as LP, Transportation Problems, Network Analysis, Simulation, Queuing Theory and other techniques. These computer packages also can calculate the range of optimality and other aspects of sensitivity analysis such as reduced cost, shadow price, and duality. These packages are different in the level of difficulty and practicality. Various available OR Software packages differ in the way of producing correct outputs. The most well-known software is the Excel solver that comes with Microsoft product. Alternatively, there are packages usually attached to most of the well-known textbooks, such as Management Scientist (An Introduction to Management Science by Anderson and others), Tora (Introduction to Operations Research by Hamdi Taha), QSB, POM, DSS, and many others. The main problem with these software packages resides in their variable trustworthiness to produce an accurate solution.

6. Conclusions

Shortcut methods were presented in this paper to produce sensitivity analysis of linear programming models. Three different topics on sensitivity were taken into account: changes in the model parameters, i.e., changes on objective function coefficients, changes on the RHS values of the LP model constraints, and calculating and interpreting the shadow price of L.P. model. The analysis has suggested a few shortcut methods to perform the sensitivity analysis that can be used in operations research and quantitative methods textbooks to be taught in educational institutes. It is very straightforward and less time is demanding to apply compared to current methods used by leading books around the world.

Attention is given to computer software packages used to solve OR Linear programming problems. These Software packages vary in setting up the input data, and also differ in the accuracy of the outputs.

The research is a significant contribution in the sense that it will assist the management and business students at different universities in making correct decisions by using very short and easy-to-calculate methods concerning the sensitivity analysis of linear programming problems.

The paper has highlighted the attention to a critical issue related to the accuracy of computer packages which are used in solving L.P. models. Some of these packages have programming errors which can not arrive at the correct answers. The author is planning to carry out research to analyze few well-known software in OR to support the investigation of such issues.

References:

Arsham, H. (1992). Post optimality analysis of the transportation problem. *The Journal of the Operational Research Society*, *43*(2), 121-139.

Anderson, D. R., Sweeney, D. J., Williams, T. A., & Wisniewski, M. (2009*). An introduction to management science: quantitative approaches to decision making*. South-Western CENGAGE Learning UK.

Bazaraa, M., & Jarvis, J. (1990). Linear Programming and Network Flows. New York: Wiley.

Baird, B. F. (1990). *Managerial Decisions Under Uncertainty. An Introduction to the Analysis of Decision Making.* New York, USA: Wiley.

Bradley, S., Hax, A., & Magnanti, T. (1977). Applied Mathematical Programming. Reading, MA: Addison-Wesley.

Bianchi, C., & Calzolari, G. (1981). A simulation approaches to some dynamic properties of econometric models. In: *Mathematical Programming and its Economic Applications*, 607-21.

Clemson, B., Tang, Y., Pyne, J., & Unal, R. (1995). Efficient methods for sensitivity analysis. *System Dynamics Review*, *11*(1), 31-49.

Dantzig, G. B. (1963*). Linear Programming and Extensions. A report prepared for United States Air Force Projected Rand.* Retrieved from http://www.rand.org/content/dam/rand/pubs/reports/2007/R366part1.pdf

Dantzig, G. B. (1978). *Are dual variables prices? If not, how to make them more so. Technical report.* Systems Optimization Laboratory, Department of Operations Research Stanford University.USA

Eschenbach, T. G. & McKeague, L. S. (1989). Exposition on using graphs for sensitivity analysis. The Engineering Economist, *34*(4), *315-333.*

Gal, T. (1979). *Postoptimal Analysis, Parametric Programming, and Related Topics*. New York, USA: McGraw-Hill

Gal, T., & Greenberg, H. J. (1997). *Advances in sensitivity analysis and parametric programming.* Boston: Kluwer.

Gass, S. (1985). *Linear Programming: Methods and Applications*, 5th ed. New York: McGraw-Hill

Hamby, D. M. (1994). A review of techniques for parameter sensitivity analysis of environmental models. *Environmental Monitoring and Assessment Journal*,*32*, 135-154.

Khan, U. I., Bajuri, H. N., & Jadoon, A. I. (2008). Optimal production planning for ICI Pakistan using linear programming and sensitivity analysis. *International Journal of Business and Social Science*, *2*(23), 206-212.

Luenberger, D. (1984). *Linear and Nonlinear Programming*, 2d ed. Reading, Mass: Addison-Wesley.

Murty, K. (1983). Linear Programming. New York: Wiley.

Murty, K. (1995). Operations Research: Deterministic Optimization Models 1st Edition. Prentice Hall.

Taha, H. (2010). *Operations Research: An Introduction*, 9th Edition. USA.

Yang, B. H. (1990). *A study on sensitivity analysis for a non-extreme optimal solution in linear programming* (Ph.D. Thesis). Seoul National University, Republic of Korea.

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