International Journal for Quality Research 10(3) 457–470
ISSN 1800-6450

Abbas Parchami [1]
Bahram Sadeghpour Gildeh
Mashaallah Mashinchi

Article info:
Received 27.12.2014
Accepted 22.07.2016

UDC – 332.05
DOI – 10.18421/IJQR10.03-01

WHY FUZZY QUALITY?

Abstract: *Such as other statistical problems, we may confront with uncertain and fuzzy concepts in quality control. One particular case in process capability analysis is a situation in which specification limits are two fuzzy sets. In such a uncertain and vague environment, the produced product is not qualified with a two-valued Boolean view, but to some degree depending on the decision-maker strictness and the quality level of the produced product. This matter can be cause to a rational decision-making on the quality of the production line. First, a comprehensive approach is presented in this paper for modeling the fuzzy quality concept. Then, motivations and advantages of applying this flexible approach instead of using classical quality are mentioned.*

Keywords: *fuzzy quality, process capability index, quality control chart, manufacturing process*

1. Introduction

In process improvement efforts, the process capability ratio or the process capability index (PCI) is a statistical measure to estimate the capability of a manufacturing productive process where in most cases the normal distribution and a large sample size are assumed for population of data; see (Kotz and Johnson, 2002) and (Montgomery, 2005) for more details. When univariate measurements are concerned, we will denote the corresponding random variable by X. The expected value and standard deviation of X will be denoted by μ and σ, respectively. Three commonly recognized PCIs are

$$C_p = \frac{USL - LSL}{6\sigma}$$

$$C_{pk} = \frac{USL - LSL - 2\left|\mu - \frac{USL + LSL}{2}\right|}{6\sigma}$$

and

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}$$

where *USL* is the upper specification limit, *LSL* is the lower specification limit, μ is the process mean, σ is the process standard deviation and T is the target value. Introducing C_p is ascribed to Juran (1974), C_{pk} to Kane (1986), and C_p for the most part to Hsiang and Taguchi (1985).

Statistical analysis in fuzzy environments has been investigated in both theory and practice over the last decades. After the inception of the theory of fuzzy sets by Zadeh (1965), many authors have applied this theory to very different areas such as statistics, mathematics, computer sciences, optimization techniques and quality control.

[1] Corresponding author: Abbas Parchami
email: parchami@uk.ac.ir

In quality control, we may confront to uncertain concepts. One case is a situation in which upper and lower specification limits (SLs) are vague. If we introduce the vagueness into SLs and express them by fuzzy terms, we face quite new, rational and interesting situation, where the ordinary PCIs are not appropriate to estimate the capability of these manufacturing processes (Parchami *et al.*, 2016).

Some authors work on process capability indices and quality control charts in fuzzy environment, which some of them are classified in Table 1. These areas are as follows: introducing fuzzy quality and fuzzy index $C_{\tilde{p}(Y)}$ based on fuzzy quality (Yongting, 1996), comparison of C_p, C_{pk} and $C_{\tilde{p}(Y)}$ capability indices (Sadeghpour Gildeh, 2003), calculation of the fuzzy PCI when observations are fuzzy numbers (Lee, 2001), construction of membership functions for PCIs when the SLs are triangular fuzzy numbers (Parchami *et al.*, 2005), application of fuzzy PCIs in educational comparison systems (Kaya and Kahraman, 2008; Parchami and Mashinchi, 2009), introducing the fuzzy PCIs on the basis of *LR* specification limits (Moeti, 2006), construction of the fuzzy confidence interval for the fuzzy capability index (Parchami et al., 2006), testing hypothesis on the constructed membership function of C_p based on fuzzy observations (Tsai and Chen, 2006), estimation of PCIs using Buckley's fuzzy estimation approach (Parchami and Mashinchi, 2007), fuzzy estimation of the loss-based Taguchi capability index (Hsu and Shu, 2008), testing the capability of fuzzy processes when SLs are triangular fuzzy numbers (Parchami and Mashinchi, 2009), construction of *p*-charts on the basis of Yongting's fuzzy quality (Amirzadeh *et al.*, 2009), introducing fuzzy PCIs for quality control of irrigation water (Kahraman and Kaya, 2009), investigation on the effects of robustness in process capability analyses (Kaya and Kahraman, 2009), developing of fuzzy PCI for decision making problems (Kaya and Kahraman, 2010), introducing a new generation of process capability indices on the basis of fuzzy SLs (Parchami and Mashinchi, 2010), process capability analyzing based on fuzzy measurements and fuzzy control charts (Kaya and Kahraman, 2011a), working on the process capability analyses with fuzzy parameters (Kaya and Kahraman, 2011b), constructing fuzzy confidence regions for the Taguchi capability index (Ramezani *et al.*, 2011; Sadeghpour Gildeh and Asghari, 2011), monitoring process capability using the process capability plots based on fuzzy data (Sadeghpour Gildeh and Moradi, 2012), and proposing a general multivariate PCI based on fuzzy tolerance region (Moradi and Sadeghpour Gildeh, 2013). In this paper, based on the fuzzy set theory, we are going to discuss on clarify and introduce the fuzzy quality concept and mention the motivations of this concept.

The organization of this paper is as follows. After presenting some preliminaries and background on using fuzzy set concept in quality control, most of trends on statistical process control based on classical/fuzzy quality are classified by a table in Section 1. The relation between the two-valued classical quality and fuzzy quality is discussed in Section 2. An approach for modeling fuzzy quality is presented in Section 3. Benefits, merits and motivations of using fuzzy quality are discussed in Section 4. Also, the classification of most essential researches on process capability indices extension for applying in fuzzy environment is discussed in Section 5. After presenting the basic idea of the main works in Section 5, all related studies briefly overviewed in each category. Finally, a conclusion is given in the final section.

2. Fuzzy quality

If the measured quality characteristic of a product is matched to the standard, i.e. if it belongs to the specification interval

$[LSL, USL]$, then this product is considered as a non-defective one in the classic quality and otherwise it is viewed as a defective product (Kotz, 2002; Kotz and Johnson, 2002). Here *LSL* and *USL* are real numbers assigned by design engineers. But in the real world, the uncertainty is a pervasive phenomenon. Much of the information on which decisions are based is uncertain. Humans have a remarkable potency and ability to make reasonable decisions based on the information which is comprise with the vagueness, uncertainty, imprecision and/or fuzziness. Formalization of this capability, at least to some degree, is a challenge that is considered by researchers recently. An approach to handle these cases is to use fuzzy sets introduced by Zadeh (1965). So, fuzzy set theory is a useful tool for considering cases that involve such uncertainty and vagueness.

For the first time, Yongting (1996) presented the concept of "fuzzy quality" by substituting the indicator function $I_{\{x|x\in[LSL,USL]\}}$ with the membership function of the fuzzy set $\tilde{Q}$. Then $\tilde{Q}(x)$ represents the degree of conformity with standard quality (or briefly, the degree of quality) where the measured quality characteristic of a product is *x*. Note that by using fuzzy quality idea, the range of quality characteristic function will be changed from $\{0,1\}$ into $[0,1]$, see Fig. 1. Also, Yongting (1996) introduced the process capability index $C_{\tilde{P}(Y)}$ as a real number and it was used by Sadeghpour Gildeh (2003).

Table 1. Trends on statistical process control based on classical and fuzzy quality by crisp and fuzzy data

Environment	Author(s)	Trend
Classical quality and crisp observations	Juran (1974) Hsiang and Taguchi (1985) Kane (1986) Kotz (1993) Kotz and Lovelace (1998) Kotz and Johnson (2002) Montgomery (2005) Parchami and Mashinchi (2007)	Introducing Cp Introducing Cpm Introducing Cpk A book on process capability indices A book on process capability indices in theory and practice Review on process capability indices A statistical book on quality control Fuzzy estimation for PCIs
Fuzzy quality and crisp observations	Yongting (1996) Sadeghpour Gildeh (2003) Mashinchi *et al.* (2005) Parchami *et al.* (2005) Parchami *et al.* (2006) Moeti *et al.* (2006) Kaya and Kahraman (2008) Parchami and Mashinchi (2009) Kahraman and Kaya (2009) Kaya and Kahraman (2009) Amirzadeh *et al.* (2009) Kaya and Kahraman (2010) Parchami and Mashinchi (2010) Ramezani *et al.* (2011)	Fuzzy quality and analysis on fuzzy probability Comparison of Cp, Cpk and Cp-tilde PCIs An educational application of fuzzy PCIs First introducing PCIs as fuzzy numbers Fuzzy confidence regions for fuzzy PCI Cp An extension on introduced PCIs by Parchami *et al.* (2005) An application of fuzzy PCIs in teaching processes Testing the capability of fuzzy processes An application of Fuzzy PCIs for quality control of irrigation water Fuzzy robust PCIs for risk assessment of air pollution Construction of p-charts using degree of nonconformity Decision making with fuzzy process accuracy index

	Sadeghpour Gildeh and Asghari (2011) Kaya and Kahraman (2011b) Sadeghpour Gildeh and Moradi (2012)	Introducing a new generation of process capability indices based on fuzzy quality Fuzzy confidence regions for fuzzy PCI Cpm using ranking functions Fuzzy confidence regions for fuzzy PCI Cpm using a defuzzification distance Process capability analyses with fuzzy parameters Fuzzy tolerance region and PCI analysis
Classical quality and fuzzy observations	Wang and Raz (1990) Raz and Wang (1990) Tsai and Chen (2006) Hsu and Shu (2008) Moradi and Sadeghpour Gildeh (2013)	First attempt to propose fuzzy control charts Defuzzify the control lines of a control chart Making fuzzy decision to evaluate process capability index Fuzzy inference to assess index Cpm with imprecise data Contour curves and testing quality based on Buckley's estimation method for families of one-sided SLs
Fuzzy quality and fuzzy observations	Lee *et al.* (1999) Lee (2001) Gülbaya and Kahraman (2007) Gülbaya and Kahraman (2006) Kaya and Kahraman (2011a)	Fuzzy design of process tolerances to maximize PCI Estimation of Cpk using fuzzy numbers Constructing fuzzy control charts for linguistic data Development control charts based on fuzzy control limits PCIs based on fuzzy measurements and fuzzy control charts

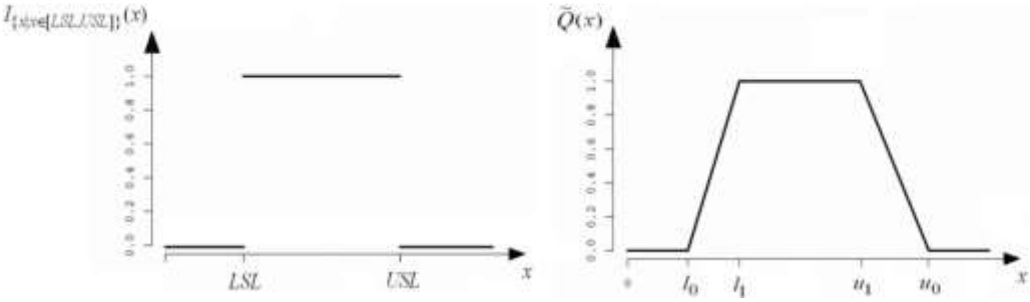

Figure 1. Characterized classical quality with the indicator function of non-defective products (left figure), and characterized fuzzy quality with the fuzzy set of non-defective products (right figure)

Amirzadeh *et al.* (2009) constructed a new control chart, called fuzzy *p*-chart, based on Yongting's fuzzy quality, and they shown that the developed control chart has a better response to variations in both the mean and the variance of the process.

Parchami and Mashinchi (2010) proved that Yongting's introduced capability index is an extension for the probability of "the product is qualified, i.e. $P(LSL < X < USL)$ in a fuzzy process. Therefore, his capability index is not a suitable extension for C_p index, since C_p is not a probability and is not always in [0,1]. Then, Parchami and Mashinchi (2010) presented a revised version of Yongting's fuzzy quality based on two fuzzy specification limits LSL and

USL which are able to characterize two non-precise concepts of “approximately bigger than” and “approximately smaller than” in a fuzzy process, respectively. An instance of fuzzy quality is characterized by two membership functions of fuzzy specification limits LSL and USL as depicted in Figure 3. Figure 2 is shown as an instance of the classical quality by characterizing two indicator functions $I_{\{x|x \geq LSL\}}$ and $I_{\{x|x \leq USL\}}$. Note that equation:

$$Q = USL \cap LSL\ ,$$

or equivalently:

$$Q(x) = \min\{USL(x), LSL(x)\}$$

presents the governed relation between membership functions of fuzzy SLs in Fig. 3 and the membership function of Yongting’s fuzzy quality in right Figure 1.

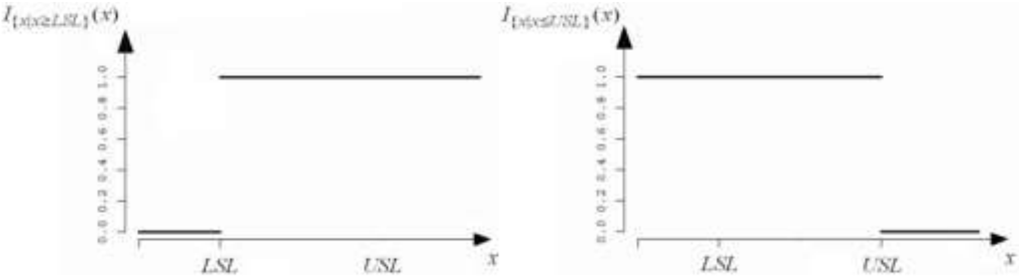

Figure 2. Characterized classical quality with two indicator functions of “bigger than *LSL*” (left figure) and “smaller than *USL*” (right figure).

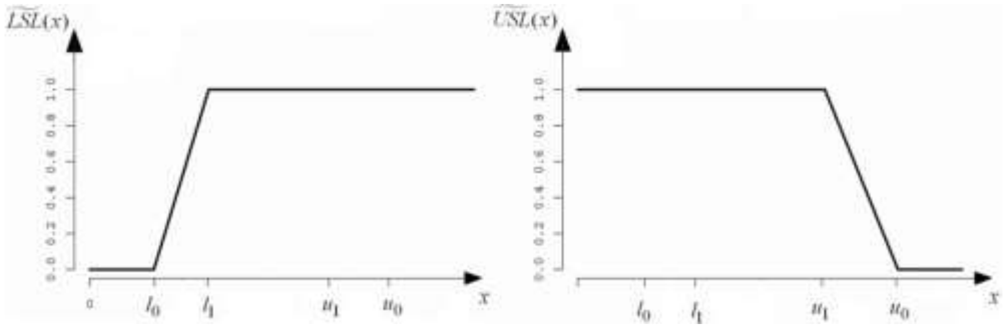

Figure 3. Characterized fuzzy quality with two membership functions of “approximately bigger than” (left figure) and “approximately smaller than” (right figure).

Similarly,

$$I_{\{x|x \in [LSL, USL]\}}(x) =$$

$$\min\{I_{\{x|x \leq USL\}}(x)\ ,\ I_{\{x|x \geq LSL\}}(x)\}$$

presents the relation between depicted indicator functions in Figure 2 and depicted indicator function of classical quality $[LSL, USL]$ in left Figure 1.

3. Modeling fuzzy quality

Let *R* be the set of all real numbers and $F(R) = \{\tilde{A} \,|\, \tilde{A} : R \rightarrow [0,1]$ is a continuous function$\}$ be the set of all fuzzy sets on *R*.

The α-cut of $\tilde{A} \in F(R)$ is the crisp set defined by $\tilde{A}_\alpha = \{x \mid \tilde{A}(x) \geq \alpha\}$, for any $\alpha \in [0,1]$. Although fuzzy SLs are introduced in (Parchami and Mashinchi, 2010), but we are going to introduce more flexible formulas for the fuzzy specification limits in follows. Let USL and LSL be two fuzzy sets with membership functions

$$USL(x) = \begin{cases} 1 & \text{if} \quad x < q \\ U(\frac{x-q}{b}) & \text{if} \quad x \geq q \end{cases}$$

and

$$LSL(x) = \begin{cases} L(\frac{p-x}{a}) & \text{if} \quad x \leq p \\ 1 & \text{if} \quad x > p, \end{cases}$$

where the reference functions $L, U: [0, +\infty] \longrightarrow [0,1]$ are non-increasing functions, $U(0) = L(0) = 1$, $p, q \in R$, and the upper and lower spreads a, $b > 0$. Then, USL and LSL are called upper fuzzy specification limit and lower fuzzy specification limit, and symbolically are denoted by $(q,b)_U$ and $(p,a)_L$, respectively. Also, $F_U(R)$ and $F_L(R)$ denote the sets of all upper and lower fuzzy specification limits, respectively.

For instance, the reference function L (or similarly U) can be considered as follow:

1) $L(x) = 1$ for $x \in [0,1]$ and 0 outside,
2) $L(x) = \max\{0, 1 - |x|^r\}$,
3) $L(x) = e^{-|x|^r}$,
4) $L(x) = \dfrac{1}{1+|x|^r}$,
5) $L(x) = \dfrac{1}{1+r|x|}$.

in which $r > 0$. It must be noted that when the spreads a and b increase, $(p,a)_L$ and $(q,b)_U$ become more and more fuzzy, respectively.

4. The motivations of fuzzy quality

In the sequel the motivations and merits of using fuzzy quality by considering fuzzy specification limits LSL and USL, instead of applying classical quality is mentioned:

1) In fuzzy quality there are more than two alternates than defective and non-defective cases. For example, a specific product may be belonging to the fuzzy set of non-defective products with a degree of say 0.7. In other words, in fuzzy quality the product is not qualified with a two valued Boolean view, but to some degree depending on the quality level of the product and the strictness of the decision maker.
2) As shown in Figure 1, in the classical quality, if the measured quality characteristic of a product is $x = LSL + \varepsilon$ ($x = LSL - \varepsilon$), then it is classified as non-defective (defective) one for a positive small number ε. But in fuzzy quality case, the lower specification limit can be chosen as a lower fuzzy limit such that there is a small significant distance between the membership values of $l_0 + \varepsilon$ and $l_0 - \varepsilon$. This means $|LSL(l_0 + \varepsilon) - LSL(l_0 - \varepsilon)|$ increases gradually as ε dose so and hence it is not an eruption in the boundaries as it behaves in classical quality.
3) Following (i) and (ii) and by substituting of the classical quality with fuzzy quality, one can expect a

justified judgment in decision making on manufacturing processes.

4) Large and complex organizations need to continuously adapt to changes in the global environment, economies, and markets. This adaptation requires solutions that consider the role of organizations, knowledge, information, processes, strategy, and technology (Kaya and Kahraman, 2011b). In recent years, some papers have been published on different areas of PCIs which have considered specification limits as fuzzy numbers; see (Kahraman and Kaya, 2009; Kaya and Kahraman, 2008; Kaya and Kahraman, 2009; Kaya and Kahraman, 2010; Kaya and Kahraman, 2011a; Kaya and Kahraman, 2012b; Mashinchi *et al.*, 2005; Moeti *et al.*, 2006; Parchami and Mashinchi, 2009; Parchami *et al.*, 2009; Ramezani *et al.*, (2011). But in this paper we use a more logical concept called fuzzy limits rather than fuzzy numbers, since the actual meanings of *USL* and *LSL* in classical quality are respectively "smaller than *USL*" and "bigger than *LSL*" which are shown by two indicator functions in Figure 2. Unlike fuzzy numbers, fuzzy limits are able to inspire the concepts of "approximately smaller than" and "approximately bigger than" as depicted membership functions of LSL and USL in Figure 3.

5) This approach increase the ability of engineers to define the specification limits (*LSL* and *USL*). Since, they can accept the products with the characteristic measures lower than *LSL* or upper than *USL* with a degree between [0,1) as up to standard (conformable) products. Therefore, another advantage of this approach is minimizing the number of non-conform items and then decreasing product costs. For example, consider the applied example given by Pearn and Chen (1997) in which the observed data have been collected from a rubber-edge production line in the factory. The upper specification limit, *USL*, and the target value, *T*, are determined to be 8.94 and 8.7, respectively. In traditional method the products with characteristic measures between 8.94 and $8.94+\varepsilon$, considered as non-conform and waste items. But, by introducing the fuzzy quality, producer can accept these items as up-to standard with an acceptance degree between [0,1).

5. Different approaches for capability indices extension in fuzzy environment

In this section, four basic methods on generalization of process capability indices for fuzzy environment are reviewed and discussed.

5.1. Lee et al.'s method

Lee *et al.* (1999) generalized the capability index C_p by extension principles based on fuzzy specifications and fuzzy data. Under a similar conditions, Lee (2001) follows his approach to generalize capability index C_{pk}. Based on triangular fuzzy observations $\tilde{x}_j = T(o_j, p_j, q_j) \in F_T(R)$, $j = 1,...,n$, and considering triangular fuzzy target value $\tilde{t} = T(w, y, z) \in F_T(R)$ and also triangular fuzzy specification limits $LSL = T(l, m, n) \in F_T(R)$ and $USL = T(o, p, q) \in F_T(R)$, Lee proposed the following approximation for the membership function of C_{pk} index

$$U_{C_{pk}}(I) = \begin{cases} \dfrac{-B_1}{2A_1} + \left[\left(\dfrac{B_1}{2A_1} \right)^2 - \dfrac{C_1 - I}{A_1} \right]^{1/2} & if \ \ C_1 \le I \le C_3 \\ \dfrac{B_2}{2A_2} - \left[\left(\dfrac{B_2}{2A_2} \right)^2 - \dfrac{C_2 - I}{A_2} \right]^{1/2} & if \ \ C_3 \le I \le C_2 \\ 0 & elsewhere, \end{cases}$$

in which:

$A_1 = (b-a)(e-d)$, $A_2=(c-b)(f-e)$,
$B_1 = a(e-d)+d(b-a)$
$B_2 = c(f-e)+f(c-b)$

$C_1 = ad$, $C_2 = cf$, $C_3 = be$

$$a = 1 - \left(\frac{\sum_{j=1}^{n} o_j}{n} - z \right) \left(\frac{2}{q-l} \right),$$

$$b = 1 - \left(\frac{\sum_{j=1}^{n} p_j}{n} - y \right) \left(\frac{2}{p-m} \right),$$

$$c = 1 - \left(\frac{\sum_{j=1}^{n} q_j}{n} - w \right) \left(\frac{2}{o-n} \right),$$

$$d = (o-n) \left(\frac{1}{6C_2} \right),$$

$$e = (p-m) \left(\frac{1}{6C_3} \right),$$

$$f = (q-l) \left(\frac{1}{6C_1} \right).$$

After computing the membership function of fuzzy PCI, he fuzzified the proposed fuzzy PCI for making final decision in the examined manufacturing process. The major advantage of the proposed method is using extension principle approach. Complex calculations, low speed of process and presenting non-exact approximates for capability indices are weakness points of Lee's method which cause increasing the progress of the proposed method.

A similar approach to solve this problem based on extension principle presented by Shu and Wu (2009) by fuzzy data. In their approach, which is easier and fasten than Lee's method, the α-cuts of fuzzy index C_{pk} was calculated based on the α-cuts of fuzzy data for $0 \le \alpha \le 1$. Meanwhile, they investigated on the capability of the LCD monitors assembly line using their generalized indices. In this regard, the capability test on the generalized capability index C_p with fuzzy data have been investigated by Tsai and Chen (2006).

5.2. Parchami et al.'s method

A process with fuzzy specification limits, which Parchami *et al.* (2005) called a fuzzy process for short, is one which approximately satisfies the normal distribution condition and its specification limits are fuzzy. They extend the classical PCIs by extension principle for fuzzy processes. For example, their extended version of C_{pm} index is

$$\tilde{C}_{pm} = T\left(\frac{a_u - c_l}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{b_u - b_l}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{c_u - a_l}{6\sqrt{\sigma^2 + (\mu - T)^2}} \right)$$

where T is target value, $a_u \ge c_l$ and the fuzzy numbers $U(a_u, b_u, c_u) = T(a_u, b_u, c_u) \in F_T(R)$ and $L(a_l, b_l, c_l) = T(a_l, b_l, c_l) \in F_T(R)$ are the upper and lower engineering specification limits, respectively. It is obvious that their extended indices are exactly triangular fuzzy numbers and they are applied when the data are crisp and specification limits are two triangular fuzzy numbers.

In recent years, some papers have been concentrated on different statistical fields of fuzzy process which we briefly review them. Moeti *et al.* (2006) introduced the fuzzy process capability indices based on *LR* specification limits. Ramezani *et al.* (2011) and Parchami *et al.* (2006; 2011) constructed several fuzzy confidence regions for fuzzy PCIs $\tilde{C}_p$ and $\tilde{C}_{pm}$, respectively. Testing the capability of fuzzy processes are investigated by Parchami and Mashinchi (2009) where specification limits are triangular fuzzy numbers. Extending other classical and conventional PCIs are followed by Kaya and Kahraman (2010) based on this method. Also, after extending this method by Kaya and Kahraman (2008) for trapezoidal fuzzy specification limits, they applied their extended PCIs to compare several educational and teaching processes (also see Mashinchi *et al.* (2005)).

5.3. Kaya and Kahraman's method

An another prevalent method for PCIs estimation is constructed on the basis of Buckley's estimation approach. Buckley (2004; 2006) propose a general estimation approach to estimate any unknown parameter by a triangular shaped fuzzy number whose α-cuts are equal to the $100(1-\alpha)\%$ confidence intervals of the parameter. Recently, several authors used Buckley's estimation approach to PCIs estimation by a triangular shaped fuzzy number when both specifications and data are crisp. Parchami and Mashinchi (2007) estimated classical PCIs C_p, C_{pk} and C_{pm} by Buckley's approach and they proposed a method for the comparison of the estimated PCIs. For instance in their approach, the α-cut of the fuzzy estimation for C_p is equivalent to

$$\left[\hat{C}_p\sqrt{\frac{\chi^2_{n-1,\alpha/2}}{n-1}}, \hat{C}_p\sqrt{\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}}\right], \; 0<\alpha<1, \tag{1}$$

in which $\hat{C}_p = \frac{USL-LSL}{6s}$ is the point estimation of C_p and $\chi^2_{n,\alpha}$ is the α-quantile of Chi-square distribution with n degrees of freedom. So, the proposed estimations for PCIs contain both point and interval estimates and so provide more information for the practitioner. Kahraman and Kaya (2009) introduced fuzzy PCIs for quality control of irrigation water. Wu (2009) proposed an approach for testing process performance C_{pk} based on Buckley's estimator with crisp data and crisp specification limits. Also, after introducing Buckley's fuzzy estimation for capability index, Wu and Liao (2009) investigated on testing process yield assuming fuzzy critical value and fuzzy p-value. It must be clarified that both data and specification limits have considered crisp in two recent works and the presented concepts are also illustrated in a case study on the light emitting diodes manufacturing process. In this regard, Kaya and Kahraman (2009) introduced fuzzy robust capability indices and they evaluated the air pollution's Istanbul by their fuzzy PCIs. For instance, the α-cut of the presented fuzzy estimation in Eq. (1) modified in their method as follows

$$\left[\hat{C}_p\sqrt{\frac{\chi^2_{n-1,\alpha/2}}{n-1}} + \left(\hat{C}_p - \hat{C}_p\sqrt{\frac{\chi^2_{n-1,0.5}}{n-1}}\right), \right.$$
$$\left. \hat{C}_p\sqrt{\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}} + \left(\hat{C}_p - \hat{C}_p\sqrt{\frac{\chi^2_{n-1,0.5}}{n-1}}\right)\right], \; 0<\alpha\le 1.$$

As another work on this topic, Hsu and Shu (2008) studied on fuzzy estimation of capability index C_{pm} to assess manufacturing process capability with imprecise data. Kaya and Kahraman (2011b) estimated classical capability indices via triangular shaped fuzzy numbers by replacing Buckley's fuzzy estimations of process mean and process standard deviation. Analyzing fuzzy PCIs followed by Kaya and Kahraman (2011a) based on fuzzy measurements and also they drawn fuzzy control charts for fuzzy measurements. Moradi and Sadeghpour Gildeh (2013) worked on fuzzy one-sided process capability plots for the family of one-sided specification limits.

5.4. Yongting's method

As presented earlier, Yongting (1996) for the first time defines fuzzy quality by substituting the indicator function $I_{\{x|x\in[LSL,USL]\}}$ with the membership function of the fuzzy set $\tilde{Q}$, where the membership function $\tilde{Q}(x)$ represents the degree of conformity of the measured quality characteristic with standard quality. Also, he introduced the capability index

$$C_{\tilde{p}(Y)} = \begin{cases} \int_{-\infty}^{+\infty} Q(x) f(x) dx & \text{continuous random variable} \\ \sum_{i=1}^{N} Q(x_i) P(x_i) & \text{discrete random variable} \end{cases}$$

based on fuzzy quality for precise data in which f and P are p.d.f. and p.m.f. of the quality characteristic, respectively.

Example 1. Suppose that a random sample is taken from an assembly line of a special product under the normality assumption. The mean and standard deviation of the observed data are $\bar{x} = 0.7$ and $s = 0.15$, respectively. First, we consider a non-symmetric triangular fuzzy quality with the following membership function for product (see Figure 4)

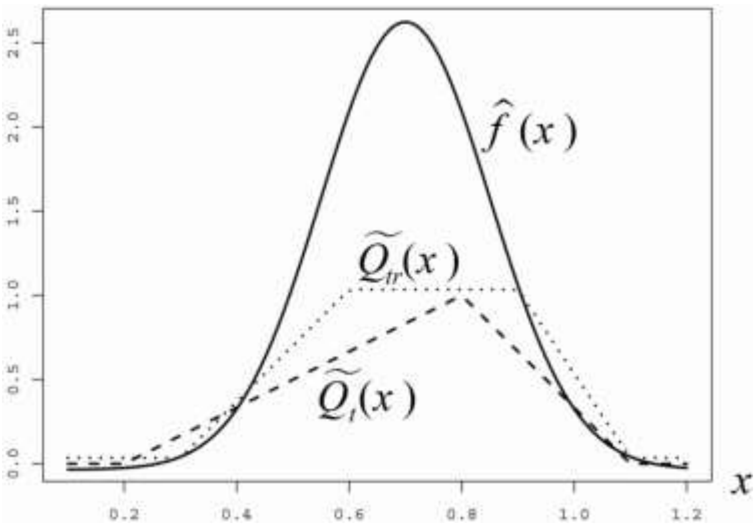

Figure 4. The membership functions of triangular and trapezoidal fuzzy qualities and the estimated probability density function of the quality characteristic in Example 1

$$Q_t(x) = \begin{cases} \dfrac{x-0.2}{0.6} & if \quad 0.2 \le x < 0.8 \\ \dfrac{1.1-x}{0.3} & if \quad 0.8 \le x < 1.1 \\ 0 & elsewhere. \end{cases}$$

In this situation, one can estimate Yongting's capability index as follow

$$\begin{aligned} C_{\tilde{p}(Y)} &= \int_{-\infty}^{+\infty} Q_t(x) f(x) dx \\ &= \frac{1}{\sqrt{2\pi}s} \int_{-\infty}^{+\infty} Q_t(x) \exp\left(-\frac{(x-\bar{x})^2}{2s^2}\right) dx \\ &= \frac{1}{0.15\sqrt{2\pi}} \left[\int_{0.2}^{0.8} \frac{x-0.2}{0.6} \exp\left(-\frac{(x-0.7)^2}{2\times 0.15^2}\right) dx \right. \\ &\qquad \left. + \int_{0.8}^{1.1} \frac{1.1-x}{0.3} \exp\left(-\frac{(x-0.7)^2}{2\times 0.15^2}\right) dx \right] \\ &= 0.543 + 0.178 = 0.721. \end{aligned}$$

Now, let us to consider a trapezoidal fuzzy quality with the following membership function for this product

$$Q_{tr}(x)=\begin{cases}\dfrac{x-0.3}{0.3} & if \quad 0.3\le x<0.6\\ 1 & if \quad 0.6\le x<0.9\\ \dfrac{1.1-x}{0.2} & if \quad 0.9\le x<1.1\\ 0 & elsewhere.\end{cases}$$

Similarly, one can estimate Yongting's capability index as follow

$$C_{\tilde{p}(Y)}=\frac{1}{\sqrt{2\pi}s}\int_{-\infty}^{+\infty}Q_{tr}(x)\exp\left(-\frac{(x-\bar{x})^2}{2s^2}\right)dx$$

$$=\frac{1}{0.15\sqrt{2\pi}}\left[\int_{0.3}^{0.6}\frac{x-0.3}{0.3}\exp\left(-\frac{(x-0.7)^2}{2\times0.15^2}\right)dx\right.$$

$$+\int_{0.6}^{0.9}\exp\left(-\frac{(x-0.7)^2}{2\times0.15^2}\right)dx$$

$$\left.+\int_{0.9}^{1.1}\frac{1.1-x}{0.2}\exp\left(-\frac{(x-0.7)^2}{2\times0.15^2}\right)dx\right]$$

$$=0.178+0.656+0.060=0.894.$$

Therefore, the process is more capable under considering the trapezoidal fuzzy quality Q_{tr} with respect to considering the triangular fuzzy quality Q_t.

See also (Sadeghpour Gildeh, 2003; Amirzadeh *et al.*, 2009; Parchami and Mashinchi, 2010), which are in the following of Yongting's method and we previously discussed on them with more details in Section 2.

6. Conclusions

Statistical quality control analysis, especially decision making on "quality concept" for each product, is often plagued by uncertainties in data, imprecision in available modeling tools and vagueness in understanding of the underlying scientific and technical underpinnings. These limitations affect regulatory and policy decisions, and must therefore be exactly communicated and measured during the decision making process for defining the quality concept by considering fuzzy sets theory. This revolution in considering "fuzzy quality" rather than two-valued classical quality is very helpful and emergent for better evaluations on some cases. After reviewing the most of trends on statistical process control based on classical/fuzzy quality in Table 1, the relation between two-valued classical quality and fuzzy quality is communicated and discussed. A general approach for modeling "fuzzy quality" is proposed with defining "fuzzy limits". This approach has the following motivations and benefits with respect to the two-valued classical quality concept: (1) considering more than two alternates defective and non-defective cases, (2) significance and meaningful difference between the quality level of defective and non-defective classes especially near the boundary of quality interval, (3) more justified judgment in decision making about manufacturing processes, (4) having the flexibility and the ability to inspire the concepts of "approximately smaller than" and "approximately bigger than" by using fuzzy limits, (5) minimizing the number of non-conform items in some cases and then decreasing product costs. Also at the end of this paper, a brief review of four basic methods for capability indices extension in fuzzy environment are presented.

References:

Amirzadeh, V., Mashinchi, M., & Parchami, A. (2009). Construction of p-charts using degree of nonconformity, *Information Sciences*, *179*(1-2), 150-160.

Buckley, J.J. (2004). *Fuzzy statistics, studies in fuzziness and soft computing*. Springer, New York.

Buckley, J.J. (2006). *Fuzzy Probability and Statistics*. Berlin, Heidelberg: Springer-Verlag.

Gülbaya, M., & Kahraman, C. (2006). Development of fuzzy process control charts and fuzzy unnatural pattern analyses. *Computational Statistics & Data Analysis, 51*, 434-451.

Gülbaya, M., & Kahraman, C. (2007). An alternative approach to fuzzy control charts: Direct fuzzy approach. *Information Sciences*, *177*, 1463-1480.

Hsiang, T.C., & Taguchi, G. (1985). *Tutorial on quality control and assurance-The Taguchi methods*. Joint Meetings of the American Statistical Association, Las Vegas, Nevada, 188.

Hsu, B.M., & Shu, M.H. (2008). Fuzzy inference to assess manufacturing process capability with imprecise data. *European Journal of Operational Research*, *186*(2), 652-670.

Juran, J.M. (1974). *Juran's Quality Control Handbook. 3rd edition*, McGraw-Hill, New York.

Kahraman, C., & Kaya, I. (2009). Fuzzy process capability indices for quality control of irrigation water. *Stochastic Environmental Research and Risk Assessment*, *23*, 451-462.

Kane, V.E. (1986). Process capability indices. *Journal of Quality Technology*, *18*, 41-52.

Kaya, I., & Kahraman, C. (2008). Fuzzy process capability analyses: An application to teaching processes. *Journal of Intelligent & Fuzzy Systems*, *19*, 259-272.

Kaya, I., & Kahraman, C. (2009). Fuzzy robust process capability indices for risk assessment of air pollution. *Stochastic Environmental Research and Risk Assessment*, *23*, 529-541.

Kaya, I., & Kahraman, C. (2010). Development of fuzzy process accuracy index for decision making problems. *Information Sciences*, *180*, 861-872.

Kaya, I., & Kahraman, C. (2011a). Process capability analyses based on fuzzy measurements and fuzzy control charts. *Expert Systems with Applications*, *38*, 3172-3184.

Kaya, I., & Kahraman, C. (2011b). Process capability analyses with fuzzy parameters. *Expert Systems with Applications*, *38*(9), 11918-11927.

Kotz, S. (1993). *Process Capability Indices*. Chapman and Hall, New York.

Kotz, S., & Johnson, N. (2002). Process capability indices- a review, 1992-2000. *Journal of Quality Technology*, *34*, 2-19.

Kotz, S., & Lovelace, C.R. (1998). *Process Capability Indices in Theory and Practice*. Oxford University Press Inc., New York.

Lee, H. T. (2001). C_{pk} index estimation using fuzzy numbers. *European Journal of Operational Research*, *129*, 683-688.

Lee, Y.H., Wei, C.C., & Chang, C.L. (1999). Fuzzy design of process tolerances to maximise process capability. *International Journal of Advanced Manufacturing Technology*, *15*, 655-659.

Mashinchi, M., Parchami, A., & Maleki, H.R. (2005). *Application of fuzzy capability indices in educational comparison*, Proceedings of 3rd International Qualitative Research Convention, Sofitel Palm Ressort, Senai, Johor, Malaysia, 21-23 August 2005, Paper No. 42.

Moeti, M.T., Parchami, A., & Mashinchi, M. (2006). A note on fuzzy process capability indices. *Scientia Iranica*, *13*(4), 379-385.

Montgomery, D.C. (2005). *Introduction to Statistical Quality Control*. 5th edition, John Wiley & Sons, New York.

Moradi, V., & Sadeghpour Gildeh, B. (2013), Fuzzy process capability plots for families of one-sided specification limits. *Advanced Manufacturing Technology*, *64*, 357-367.

Parchami, A., & Mashinchi, M. (2007). Fuzzy estimation for process capability indices. *Information Sciences*, *177*(6), 1452-1462.

Parchami, A., & Mashinchi, M. (2009). Testing the capability of fuzzy processes. *Quality Technology and Quantitative Management*, *6*(2), 125-136.

Parchami, A., & Mashinchi, M. (2010). A new generation of process capability indices. *Journal of Applied Statistics*, *37*(1), 77-89.

Parchami, A., & Mashinchi, M. (2011). Interval estimation of an extended capability index with application in educational systems. *Turkish Journal of Fuzzy Systems*, *2*(2), 64–76.

Parchami, A., & Sadeghpour Gildeh, B. (2016) *Trends on process capability indices in fuzzy environment*, C. Kahraman and S. Yanik (eds.), Intelligent Decision Making in Quality Management, Springer International Publishing Switzerland, Chapter 5, 127-140.

Parchami, A., Mashinchi, M., & Maleki, H.R. (2006). Fuzzy confidence intervals for fuzzy process capability index. *Journal of Intelligent & Fuzzy Systems*, *17*(3), 287-295.

Parchami, A., Mashinchi, M., Yavari, A. R., & Maleki, H. R. (2005). Process Capability Indices as Fuzzy Numbers. *Austrian Journal of Statistics*, *34*(4), 391-402.

Pearn, W. L., and Chen, K. S. (1997) A practical implementation of the process capability index C_{pk}, *Quality Engineering*, *9*(4), 721-737.

Ramezani, Z., Parchami, A., & Mashinchi, M. (2011). Fuzzy confidence regions for the Taguchi capability index. *International Journal of Systems Science*, *42*(6), 977-987.

Raz, T., & Wang, J.H. (1990). Probabilistic and membership approaches in construction of control charts for linguistic data. *Production Planning & Control*, *1*, 147-157.

Sadeghpour Gildeh, B. (2003). *Comparison of* C_P, C_{pk} *and* $\tilde{C}_P$ *process capability indices in the case of measurement error occurrence*. IFSA World Congress, Istanbul, Turkey, 563-567.

Sadeghpour Gildeh, B., & Moradi, V. (2012). Fuzzy tolerance region and process capability analysis. Advances in Intelligent and Soft Computing, 147, 183-193.

Sadeghpour Gildeh, B., Asghari, S. (2011). A new method for constructing confidence interval for C_{pm} based on fuzzy data. *International Journal for Quality Research*, *5*(2), 67-73.

Tsai, C. C., and Chen, C. C. (2006). Making decision to evaluate process capability index C_p with fuzzy numbers. *International Journal of Advanced Manufacturing Technology*, *30*, 334-339.

Wang, J. H., & Raz, T. (1990). On the construction of control charts using linguistic variables. *International Journal of Production Research*, *28*, 477-487.

Wu, C.W. (2009). Decision-making in testing process performance with fuzzy data. *European Journal of Operational Research*, *193*, 499-509.

Wu, C.W., & Liao, M.Y. (2009). Estimating and testing process yield with imprecise data. *Expert Systems with Applications*, *36*, 11006-11012.

Yongting, C. (1996). Fuzzy quality and analysis on fuzzy probability. *Fuzzy Sets and Systems*, *83*, 283-290.

Zadeh, L.A. (1965). Fuzzy sets. *Information and Control*, *8*, 338-359.

Abbas Parchami
Shahid Bahonar University of Kerman,
Department of Statistics, Faculty of Mathematics and Computer
Kerman
Iran
parchami@uk.ac.ir

Bahram Sadeghpour Gildeh
Ferdowsi University of Mashhad,
Department of Statistics, Factually of Mathematical Sciences
Mashhad
Iran
sadeghpour@um.ac.ir

Mashaallah Mashinchi
Shahid Bahonar University of Kerman,
Department of Statistics, Faculty of Mathematics and Computer
Kerman
Iran
mashinchi@uk.ac.ir