

## FINITE ELEMENT EVALUATION AND OPTIMIZATION OF GEOMETRY WITH DOE

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**Abstract:** *Since 1960, Taguchi methods have been used for improving the quality of Japanese products with great success. Basic assumption of Taguchi's design for six sigma or robust design is that quality must be designed into a product from the start at both the product and process design stage in order to improve product reliability and manufacturability. This paper deals with case study of product design based on Taguchi's approach that involves parametric optimization of piston rod geometry aiming mass reduction with stress restriction. Finite element analysis software ANSYS Workbench was used to get access to CAD parameters of piston rod within a process of parametric finite element evaluation and optimization.*

**Keywords:** *design for six sigma, finite element analysis, parametric optimization*

### 1. INTRODUCTION

Since 1960, Taguchi methods have been used for improving the quality of Japanese products with great success. Recently companies in the United States and Europe began adopting Taguchi's robust design approach in an effort to improve product quality and design robustness. Design for six sigma or robust design is an "engineering methodology for improving productivity during research and development so that high-quality products can be produced quickly and at low cost" [1]. The idea behind robust design is to improve the quality of a product by minimizing the effects of variation without eliminating the causes, since they are too difficult or too expensive to control. Taguchi method is a quality control method that is instituted at both the product and process design stage to improve product manufacturability and reliability by making products insensitive to environmental conditions and component variations. Taguchi stressed out that quality must be designed into a product from the start. The end result is a robust design, a design that has minimum sensitivity to variations in uncontrollable factors.

Taguchi bases his method on conventional statistical tools together with some guidelines for laying out design experiments and analyzing the results of these experiments. Taguchi's approach to quality control applies to the entire process of developing and manufacturing a product, from the initial concept, through design and engineering, to manufacturing and production. However building quality into the product during the design stage is the ultimate goal within the Taguchi philosophy [2].

This paper deals with case study of product design based on Taguchi's approach that involves parametric optimization of piston rod geometry aiming mass reduction with stress restriction. Finite element analysis

software ANSYS Workbench was used to get access to CAD parameters of piston rod within a process of parametric finite element evaluation and optimization.

### 2. METHODS

To achieve desirable product quality by design, Taguchi suggests a three-stage process: system design, parameter design, and tolerance design [3,4,5]. System design is the conceptualization and synthesis of a product to be used. The system design stage is where new ideas, concepts and knowledge in the areas of science and technology are utilized by the design team to determine the right combination of materials, parts, processes and design factors that will satisfy functional and economical specifications. In parameter design the system variables are experimentally analyzed to determine how the product reacts to uncontrollable "noise" in the system. Parameter design is the main thrust of Taguchi's approach. Parameter design is related to finding the appropriate design factor levels to make the system less sensitive to variations in uncontrollable noise factors, i.e., to make the system robust. The final step in Taguchi's robust design approach is tolerance design. Tolerance design occurs when the tolerances for the products are established to minimize the sum of the manufacturing and lifetime costs of the product. In the tolerance design stage, tolerances of factors that have the largest influence on variation are adjusted only if after the parameter design stage, the target values of quality have not yet been achieved. Most engineers tend to associate quality with better tolerances, but tightening the tolerances increases the cost of the product because it requires better materials, components, or machinery to achieve the tighter tolerances. Taguchi's parameter design approach allows for improving the quality without requiring

better materials or parts and makes it possible to improve quality and decrease or at least maintain the same cost.

In parameter design, there are two types of factors that affect a product’s functional characteristic: control factors and noise factors. Control factors are those factors which can easily be controlled such as dimensions, material choice or cycle time. Noise factors are factors that are difficult or impossible or too expensive to control. There are three types of noise factors: outer noise (vibration, shock, temperature...), inner noise (deterioration of parts and materials, rust...), and between product noise (Young’s modulus, shear modulus, allowable stress...) [6]. Noise factors are primarily response for causing a product’s performance to deviate from its target value. Hence, parameter design seeks to identify settings of the control factors which make the product insensitive to variations in the noise factors, i.e., make the product more robust, without actually eliminating the causes of variation.

Design of Experiments (DOE) techniques, specifically orthogonal arrays, are employed in Taguchi’s approach to systematically vary and test the different levels of each of the control factors. To implement robust design, Taguchi advocates the use of an “inner array” and “outer array” approach. The “inner array” consists of the orthogonal arrays that contain the control factor settings. The “outer array” consists of the orthogonal arrays that contain the noise factors and their settings which are under investigation. The combination of the “inner array” and “outer array” constitutes what is called the “product array” or “complete parameter design layout.” The product array is used to systematically test various combinations of the control factor settings over all combinations of noise factors after which the mean response  $\bar{y}$  and standard deviation  $S$  may be approximated for each run using the following equations:

$$\bar{y} = \frac{1}{n} \cdot \sum_{i=1}^n y_i \quad (1)$$

$$S = \sqrt{\sum_{i=1}^n \frac{(y_i - \bar{y})^2}{n-1}} \quad (2)$$

The preferred parameter settings are then determined through analysis of the “signal-to-noise” (SN) ratio where factor levels that maximize the appropriate SN ratio are optimal. There are three standard types of SN ratios depending on the desired performance response [1,3]:

- Smaller the better (for making the system response as small as possible)

$$SN_S = -10 \log \left( \frac{1}{n} \cdot \sum_{i=1}^n y_i^2 \right) \quad (3)$$

- Nominal the best (for reducing variability around a target)

$$SN_T = 10 \log \left( \frac{y}{S^2} \right)^{-2} \quad (4)$$

- Larger the better (for making the system response as large as possible)

$$SN_L = -10 \log \left( \frac{1}{n} \cdot \sum_{i=1}^n \frac{1}{y_i^2} \right) \quad (5)$$

This paper deals with parametric optimization of geometry, target was reduction of product’s mass with restriction of stress. Finite element analysis software ANSYS Workbench was used to get access to CAD parameters and stress within a parametric finite element optimization.

The results of a finite element analysis depend on several input variables, such as material properties and CAD parameters. In a design optimization based on DOE, each change of the value of any input variable requires a new finite element analysis. A response surface is generated which is an explicit approximation function of the finite element results expressed as a function of all selected input variables. The DOE technique generates a response surface using curve- and surface-fitting algorithms to “fit” output data as a function of input data. This requires a group of design points where each point is generated via a finite element solve.

ANSYS module DesignXplorer VT provides a much more efficient approach by providing a response surface that is based on a single finite element solve combined with the use of mesh morphing and the Taylor series expansion approximation [7]. Because the derivatives are also calculated, this “extended” finite element analysis may take longer than a regular solve. However, this one “extended” finite element analysis takes considerably less time compared to the many solution runs that are required for a regular DOE solve. For six sigma analysis in DesignXplorer, the sample generation is based on the Latin Hypercube Sampling (LHS) technique [7]. The LHS technique is a more advanced and efficient form of Monte Carlo simulation methods. The only difference between LHS and the direct Monte Carlo sampling technique is that LHS has a sample “memory,” meaning it avoids repeating samples that have been evaluated before (it avoids clustering samples). It also forces the tails of a distribution to participate in the sampling process. Generally, the LHS technique requires 20% to 40% fewer simulations loops than the direct Monte Carlo simulation technique to deliver the same results with the same accuracy.

In DesignXplorer, the product behavior is expressed using the chosen [response parameters](#). In this paper the maximum equivalent (von-Mises) stress and mass are used to decide whether the product behavior is acceptable. Robust design is based on assumptions regarding scatter, or uncontrollable uncertainties.

Scatter in the input parameters (control factors) that affect the response parameters (noise factors) will cause the response parameters to also be uncertain and therefore less predictable.

Robust design that is interpreted from a six sigma analysis leads to an optimization problem that tries to achieve or enforce a design that satisfies six sigma analysis quality goals. Therefore, before one runs a robust design, one must first [parameterize the results](#) of a six sigma analysis. In this paper SN ratio is parameterized.

The goal of a robust design is to maximize the appropriate SN ratio of a response parameter. The parameter settings that maximize the appropriate SN ratio is a Pareto optimal solution of a product parametric optimization.

### 3. DESIGN OF EXPERIMENT

A piston rod, chosen for parametric optimization of geometry, is loaded in axial direction by cyclic force with magnitude of (0÷90) kN. Endurance limit under repeated cyclic loading of steel Č1330, that piston rod is made of, is (280÷330) MPa [8].

#### 3.1. Geometric modeling

ANSYS module DesignModeler was used to generate (figure 1) and to parametrize (figure 2) geometric model of piston rod. To avoid regeneration failures mathematical relations were created between parameters and other dimensions by means of the DesignModeler parameter manager.

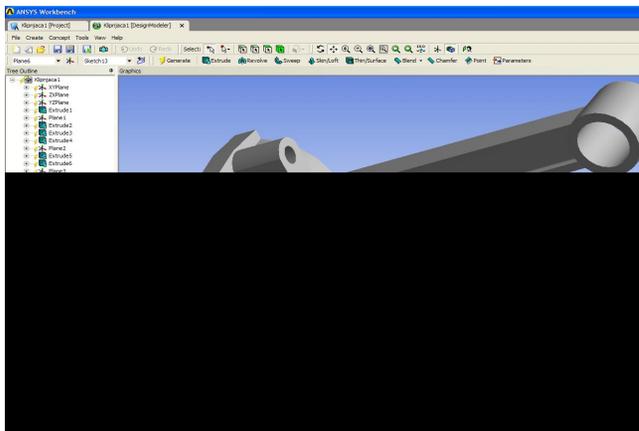


Figure 1 – Geometric model of piston rod in ANSYS DesignModeler environment

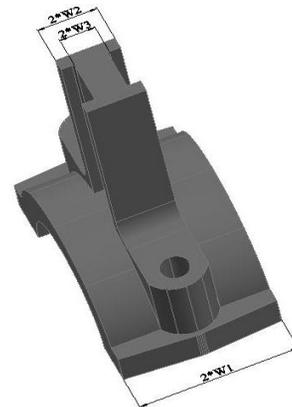


Figure 2 – Geometric model parameters

#### 3.2. Meshing

The meshing strategy for the optimization is not exactly the same as for a single finite element analysis. The mesh for the optimization task has to meet four different requirements [8,9]:

- Automated meshing must be possible for changing edges, angles and surfaces;
- The mesh quality must be comparable for every parameter combination;
- Accurate results for the changing geometry;
- In light of the expected number of calculations the calculation time should not be too long.

For a single finite element analysis it would be possible to locate problematic regions with high stress gradients and to refine the mesh at these specific regions. But concerning the second requirement this is not possible for the optimization model because critical regions as well as maximal stress can change the location due to

parameter variation. At this point sensitivity analysis was started to unveil some important features of the finite element model: are there any regeneration problems, can the ANSYS Workbench mesher always find a mesh, where are the maximal stresses located. With ANSYS module Simulation moderate changes in the geometry within the variation range of parameters were made, many meshes with different element size were generated and finite element analyses were made for many models with different geometry and different mesh. Results of this analyses were used to compare nodal solution with element solution to get an idea of result quality. At the end the final mesh with the following characteristics was generated: a general element size for the model was 2 mm, the relevance for the model was set to 100 that is the highest level and the shape checking mode was set to aggressive. The final model was meshed with 164403 nonlinear tetrahedral elements with 242697 nodes (figure 3).

### 3.3. Material properties and boundary conditions

Piston rod material was assumed to be homogenous, isotropic and linear elastic with a Poisson's ratio of 0.3 and an elastic modulus of  $2.1 \cdot 10^5$  MPa. The model is

loaded with an axial force on the crankshaft mounting ring (figure 4.). Fixed support was used to model the support at the pin hole and cylindrical supports were used to model the supports at the bolt holes (figure 4.).

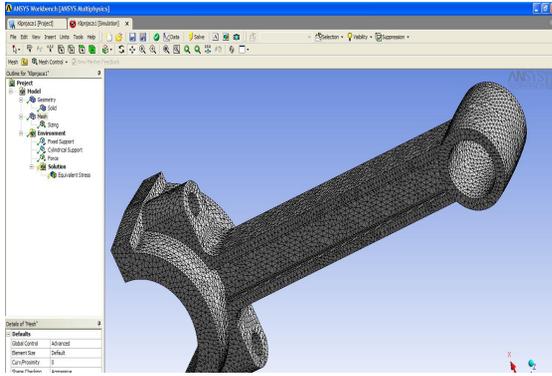


Figure 3 – Meshed model of piston rod in ANSYS Simulation environment

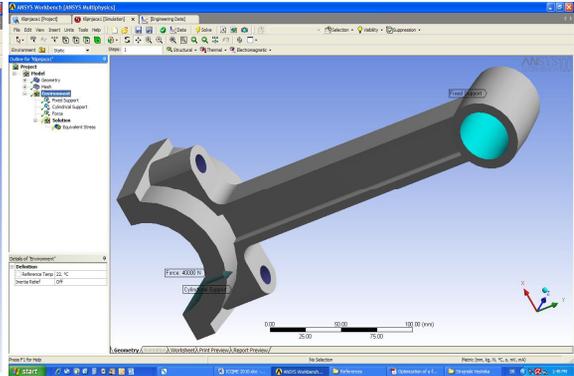


Figure 4 – Boundary conditions of the model in ANSYS Simulation environment

### 3.4. Optimization workflow

After generating an accurate finite element model a strategy for the optimization workflow was defined. Target of the optimization was to reach a mass reduction of the piston rod with the following restrictions:

- Maximal equivalent (von-Mises) stress has to be less than minimal endurance limit under repeated cyclic loading of 280 MPa;

- The variation ranges of design parameters were  $24.5 \leq w_1$  [mm]  $\leq 36$ ,  $11 \leq w_2$  [mm]  $\leq 13.5$  and  $10 \leq w_3$  [mm]  $\leq 15$ .

Finite element analysis for initial design of piston rod ( $w_1=36$  mm,  $w_2=13.5$  mm,  $w_3=15$  mm and a mass of 3.02 kg) and maximal axial force was made and maximal equivalent (von-Mises) stress (figure 5) and mass were set to be the response parameters for checking the model behavior.

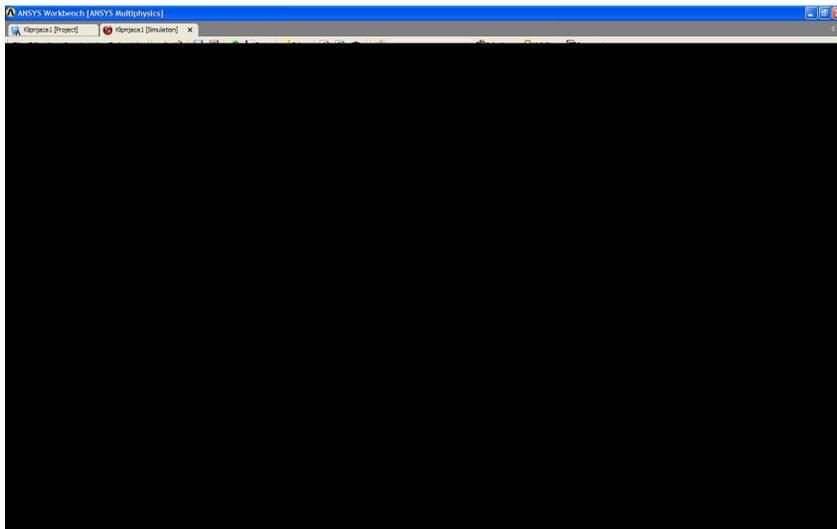


Figure 5 – Von-Mises stress distribution of the model in ANSYS Simulation environment

ANSYS module DesignXplorer VT was used to define input parameters for robust design. Parameters  $w_1, w_2$  and  $w_3$  were defined as design variables and axial force

was defined as uncertainty variable with Gaussian distribution (figure 6).

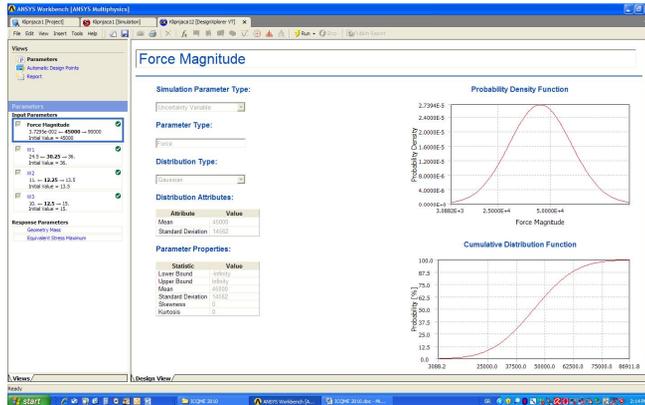


Figure 6 – Definition of input parameters for robust design in ANSYS DesignXplorer VT environment

Afterwards a six sigma analysis of the model was performed and [parameterization the results](#) of a six sigma analysis was made in order to define additional optimization targets for robust design.

#### 4. RESULTS

Based on input parameters a six sigma analysis, for sample set with 1000 design points within design space of the model, was made in order to determine distributions of the response parameters (figure 7.).

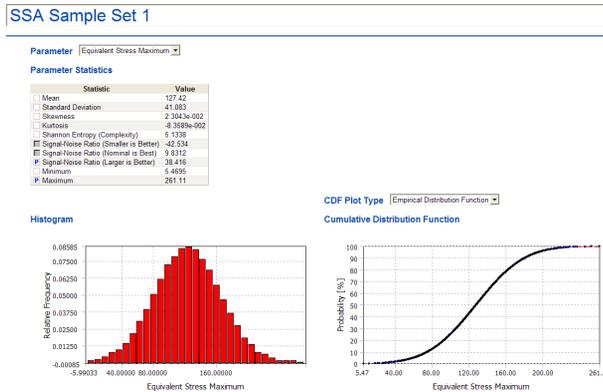


Figure 7 – Distribution of maximal equivalent stress in ANSYS DesignXplorer VT environment

Afterwards SN ratio larger the better [4,7] and maximum of distribution of maximal equivalent stress were parametrized. Another target of the optimization is to maximize parametrized SN ratio of distribution of maximal equivalent stress.

ANSYS module DesignXplorer VT was used to define targets and restrictions for robust design (figure 8.) and a robustness analysis, for sample set with 1000 design points within design space of the model, was made.

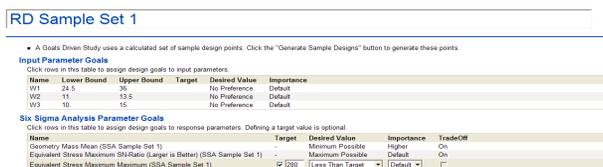


Figure 8 – Definition of targets and restrictions for robust design in ANSYS DesignXplorer VT environment

Based on the results of a robustness analysis a sensitivity analysis of influence of input parameters to response parameters was made. The

most significant input parameter regarding its influence to model mass is design parameter  $w_1$  (figure 9).

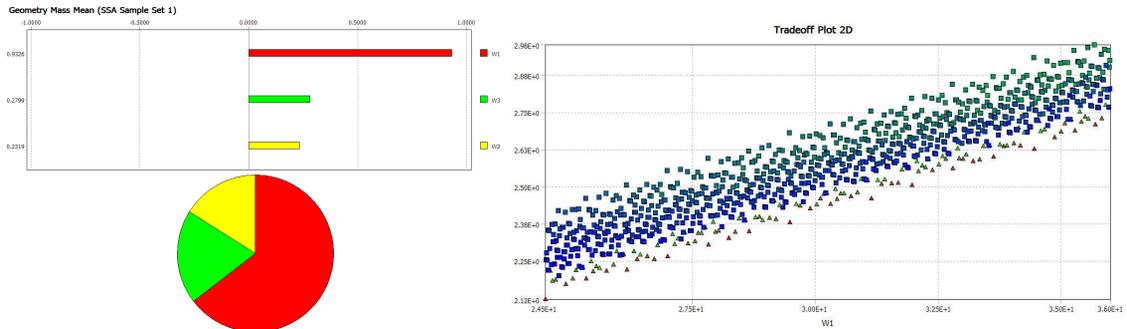


Figure 9 – Results of sensitivity analysis of model mass to input parameters

The most significant input parameter regarding its influence to model maximal equivalent stress  $S/N$

ratio is design parameter  $w_3$  (figure 10).

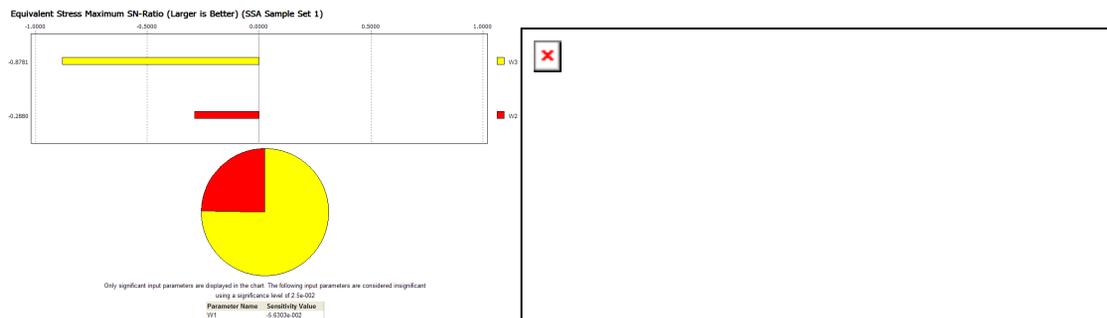


Figure 10 – Results of sensitivity analysis of maximal equivalent stress  $S/N$  ratio to input parameters

The most significant input parameter regarding its influence to model maximal equivalent stress maximum

is design parameter  $w_3$  (figure 11).

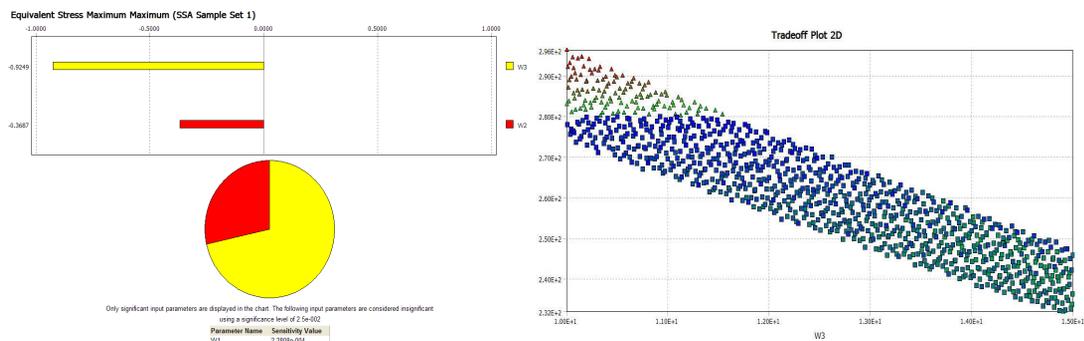


Figure 12 – Results of sensitivity analysis of maximal equivalent stress maximum to input parameters

A model with a mass of 2.2 kg, maximal equivalent (von-Mises) stress of 279.1 MPa and design variables  $w_1 \approx 24.8$  mm,  $w_2 \approx 11.2$  mm and  $w_3 \approx 11.5$  mm (figure 13)

was found as a Pareto optimal solution (figure 14). Compared to the initial model design, mass reduction of 27.2% was achieved

RD Sample Set 2

• A Goals Driven Study uses a calculated set of sample design points. Click the "Generate Sample Designs" button to generate these points.

Click rows in this table to assign design goals to input parameters.

Name	Lower Bound	Upper Bound	Target	Desired Value	Importance
W1	24.5	35	No Preference	Default	
W2	11	33.5	No Preference	Default	
W3	15	15	No Preference	Default	

Six Sigma Analysis Parameter Goals

Click rows in this table to assign design goals to response parameters. Defining a target value is optional.

Name	Target	Desired Value	Importance	TradeOff
Geometry Mass Mean (SSA Sample Set 1)	...	Minimum Possible	Higher	On
Equivalent Stress Maximum (Larger is Better) (SSA Sample Set 1)	...	Maximum Possible	Default	On
Equivalent Stress Maximum Maximum (SSA Sample Set 1)	293	Less Than Target	Higher	Off

Candidate Designs

Generate or update candidate designs based on the current goals

Parameter	Candidate A	Candidate B	Candidate C
W1	24.65	24.762	27.616
W2	11.021	11.236	11.045
W3	10.893	11.488	10.441
Geometry Mass Mean (SSA Sample Set 1)	2.1190	2.2093	2.303
Equivalent Stress Maximum (Larger is Better) (SSA Sample Set 1)	40.266	39.303	39.983
Equivalent Stress Maximum Maximum (SSA Sample Set 1)	296.19	279.95	291.95

Figure 13 – Results of robustness analysis in ANSYS DesignXplorer VT environment



Figure 14 – Pareto optimal solution of model design with distribution of Von-Mises stress

## 5. CONCLUSION

Final design of piston rod with mass reduction of 27.2% is achieved during the design stage by means of design approach based on design for six sigma combined with finite element analysis. At the same time this product design satisfies all design restrictions. Obviously this

approach leads to better quality of product design. Therefore design for six sigma combined with finite element analysis is very powerful design tool for all engineers who aim to building quality into the product during the design stage that is the ultimate goal within the Taguchi philosophy.

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